

NONLINEAR FIXED-ORDER DYNAMIC COMPENSATION FOR PASSIVE SYSTEMS[†]

WASSIM M. HADDAD* AND VIJAYA-SEKHAR CHELLABOINA

School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, U.S.A.

SUMMARY

A method for designing nonlinear fixed-order (i.e. full- and reduced-order) dynamic passive controllers for passive systems is developed using nonlinear dissipation theory. Specifically, by extending linear passive controller synthesis frameworks a family of globally asymptotically stabilizing nonlinear passive controllers is developed that can serve to enhance system performance and energy dissipation. The proposed approach is applied to the rotational/translational proof-mass actuator (RTAC) nonlinear benchmark problem to develop fixed-order dynamic output feedback nonlinear controllers. © 1998 John Wiley & Sons, Ltd.

Key words: nonlinear control; fixed-order dynamic compensation; passive systems; Lyapunov functions; nonlinear benchmark problem

1. INTRODUCTION

In certain applications, such as the colocated control of flexible structures, the system dynamics are known to be passive. While modern linear and nonlinear control design techniques can be used to control such systems when the plant dynamics are actually known, in practice the inherent robustness properties of passive controllers are often desirable when the system dynamics are uncertain. Specifically, the prospects of controlling uncertain passive systems is quite good since, if sensor and actuator dynamics are neglected, robust stability is unconditionally guaranteed so long as the controller is passive.^{1–8}

In this paper we develop a framework for designing nonlinear fixed-order (i.e. full- and reduced-order) dynamic output feedback passive controllers for nonlinear passive systems. In particular, using nonlinear dissipation theory,^{9–12} we develop a family of nonlinear passive controllers by modifying a given linear positive real compensator. These nonlinear controller modifications can serve to enhance system performance and energy dissipation while maintaining stability robustness. Furthermore, since our approach builds upon linear full- and reduced-order positive real controllers it can be viewed as a nonlinear extension of the linear passive controller synthesis frameworks presented in References 1–8.

It is shown that the nonlinear passive controller synthesis framework originally proposed by Hyland¹³ and further explored in Reference 14 to significantly enhance energy flow between plant

* Correspondence to: W. M. Haddad, School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, U.S.A. E-mail: wm.haddad@aerospace.gatech.edu.

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and controller by introducing nonlinearities that induce broad-band spectral properties in the controller is a special case of the nonlinear passive controller parameterization proposed in this paper. In particular, for control of collocated flexible structures, it was shown in Reference 13 that if low-frequency energy is injected into the system it can be dispersed to higher-frequency bands by virtue of the coupling among the vibrational modes of the structure. This energy is transferred to high frequencies and dissipated into heat through natural structural damping and the nonlinear passive controller. Hence, a purposefully designed nonlinear passive controller could create higher-frequency harmonics and effectively transfer low-frequency disturbances to high-frequency bands and hence maximize system dissipation.

Using the proposed nonlinear passive controller synthesis framework, full- and reduced-order dynamic output feedback controllers are designed for the benchmark nonlinear control design problem proposed in Reference 15. This problem was originally studied as a simplified model of a dual-spin spacecraft to investigate the resonance capture phenomenon¹⁶ and more recently was studied to investigate the utility of a rotational proof-mass actuator for stabilizing translational motion.^{17–19} The problem is interesting since it exhibits nonlinear coupling between the translational and rotational motions.

The contents of the paper are as follows. In Section 2 we provide definitions and mathematical preliminaries. Then, in Section 3, we develop a framework for designing nonlinear fixed-order dynamic passive controllers for nonlinear passive systems. In Section 4 we specialize the results of Section 3 to linear positive real systems and provide a framework for designing positive real controllers using an algebraic Riccati equation approach. In Section 5 we generalize the results of Section 4 to full- and reduced-order passive nonlinear controller synthesis by introducing nonlinear modifications to passive linear controllers for enhanced energy dissipation. Section 6 applies the framework developed in Section 5 to the rotational/translational proof-mass actuator (RTAC) nonlinear benchmark control design problem. Finally, we draw conclusions in Section 7.

2. MATHEMATICAL PRELIMINARIES

In this section we establish definitions, notation and several key results used later in the paper. Let \mathbb{R} and \mathbb{C} denote the real and complex numbers, let $()^T$ and $()^*$ denote transpose and complex conjugate transpose, respectively, and let I_n or I denote the $n \times n$ identity matrix. Furthermore, $M \geq 0$ (resp., $M > 0$) denotes the fact that the Hermitian matrix M is non-negative (resp., positive) definite. Let

$$G(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

denote a state-space realization of a transfer function $G(s)$, i.e. $G(s) = C(sI - A)^{-1}B + D$. The notation ' \sim ' is used to denote a minimal realization.

A square transfer function $G(s)$ is called *positive real*²⁰ if (1) $G(s)$ is stable, and (2) $G(s) + G^*(s)$ is non-negative definite for all $\text{Re}[s] > 0$. A square transfer function $G(s)$ is called *strictly positive real*²⁰ if $G(s - \varepsilon)$ is positive real for some $\varepsilon > 0$.

Next we state the well-known Kalman–Yakubovich–Popov lemma to characterize positive realness in the state-space setting.²⁰

Lemma: 2.1

The transfer function

$$G(s) \underset{\text{min}}{\sim} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

is positive real if and only if there exist matrices $P \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{p \times n}$, and $W \in \mathbb{R}^{p \times m}$ with P positive definite such that

$$0 = A^T P + PA + L^T L \tag{1}$$

$$0 = B^T P - C + W^T L \tag{2}$$

$$0 = D + D^T - W^T W \tag{3}$$

Remark 2.1

Note that (1)–(3) imply positive realness of

$$G(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

whether or not $G(s)$ is minimal.

Note that $G(s)$ is strictly positive real if and only if there exist matrices $P \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{p \times n}$, and $W \in \mathbb{R}^{p \times m}$ with P positive definite and a positive scalar ε such that (2) and (3) are satisfied and

$$0 = A^T P + PA + \varepsilon P + L^T L \tag{4}$$

In this paper we consider nonlinear systems \mathcal{G} of the form

$$\dot{x}(t) = f(x(t)) + G(x(t))u(t), \quad x(0) = x_0, \quad t \geq 0 \tag{5}$$

$$y(t) = h(x(t)) + J(x(t))u(t) \tag{6}$$

where $x \in \mathbb{R}^n$, $u, y \in \mathbb{R}^m$, $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$, $G: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$, $h: \mathbb{R}^n \rightarrow \mathbb{R}^m$, and $J: \mathbb{R}^n \rightarrow \mathbb{R}^{m \times m}$. We assume that $f(\cdot)$, $G(\cdot)$, $h(\cdot)$, and $J(\cdot)$ are smooth (C^∞ mappings) and $f(\cdot)$ has at least one equilibrium so that $f(0) = 0$ and $h(0) = 0$. Furthermore, for the nonlinear system \mathcal{G} we assume that the required properties for the existence and uniqueness of solutions are satisfied, i.e. $u(\cdot)$ satisfies sufficient regularity conditions such that system (5) has a unique solution forward in time. For the dynamical system \mathcal{G} given by (5) and (6), a function $r: \mathbb{R}^m \times \mathbb{R}^m \rightarrow \mathbb{R}$ is called a *supply rate*⁹ if it is locally integrable, i.e. for all input–output pairs $u, y \in \mathbb{R}^m$, $r(\cdot, \cdot)$ satisfies $\int_{t_1}^{t_2} |r(u(s), y(s))| ds < \infty$, $t_1, t_2 \geq 0$.

Definition 2.1 (Reference 9)

A system \mathcal{G} of the form (5), (6) is *dissipative with respect to the supply rate r* if there exists a C^0 non-negative-definite function $V_s: \mathbb{R}^n \rightarrow \mathbb{R}$, called a *storage function*, such that the *dissipation*

inequality

$$V_s(x(t)) \leq V_s(x(t_1)) + \int_{t_1}^t r(u(s), y(s)) \, ds \quad (7)$$

is satisfied for all $t_1, t \geq 0$ and where $x(t), t \geq 0$, is the solution to (5) with $u \in \mathbb{R}^m$.

If $V_s(\cdot)$ is C^1 -continuous then an equivalent statement for dissipativeness of \mathcal{G} with respect to the supply rate r is

$$\dot{V}_s(x(t)) \leq r(u(t), y(t)), \quad t \geq 0 \quad (8)$$

where $\dot{V}_s(\cdot)$ denotes the total derivative of $V_s(x)$ along the state trajectories $x(t), t \geq 0$, of (5). For particular examples of mechanical systems with force inputs and velocity outputs, we can associate the storage function with the stored or available energy in the system and the supply rate with the net flow of energy or power into the system. However, as discussed in References 9 and 11, the concepts of supply rates and storage functions also apply to more general systems for which this energy interpretation is no longer valid.

Definition 2.2 (Reference 12)

A system \mathcal{G} of the form (5), (6) is *passive* if \mathcal{G} is dissipative with respect to the supply rate $r(u, y) = u^T y$.

Definition 2.3 (Reference 12)

A system \mathcal{G} of the form (5), (6) is *input strict passive* if there exists $\varepsilon > 0$ such that \mathcal{G} is dissipative with respect to the supply rate $r(u, y) = u^T y - \varepsilon u^T u$.

Definition 2.4 (Reference 12)

A system \mathcal{G} of the form (5), (6) is *output strict passive* if there exists $\varepsilon > 0$ such that \mathcal{G} is dissipative with respect to the supply rate $r(u, y) = u^T y - \varepsilon y^T y$.

Definition 2.5 (Reference 21)

A system \mathcal{G} of the form (5), (6) is *state strict passive* if the dissipation inequality (7) is strictly satisfied with supply rate $r(u, y) = u^T y$.

Next, in order to state the nonlinear form of the Kalman–Yakubovich–Popov lemma, we require one more definition and a key lemma concerning dissipative systems.

Definition 2.6 (Reference 10)

A system \mathcal{G} is *zero-state observable* if for all $x \in \mathbb{R}^n, u(t) \equiv 0, y(t) \equiv 0$ implies $x(t) \equiv 0$. A system \mathcal{G} is *completely reachable* if for all $x(t_1), x(t_2)$ there exists a finite time t_2 and a square-integrable control $u(t)$ defined on $[t_1, t_2]$ such that the state $x(t), t \geq 0$, can be driven from $x(t_1)$ to $x(t_2)$.

Lemma 2.2 (Reference 12)

Let $Q \in \mathbb{R}^{l \times l}$, $S \in \mathbb{R}^{l \times m}$, $R \in \mathbb{R}^{m \times m}$, and let \mathcal{G} be zero-state observable and completely reachable. \mathcal{G} is dissipative with respect to the supply rate $r(u, y) = y^T Q y + 2y^T S u + u^T R u$ if and only if there exist functions $V_s: \mathbb{R}^n \rightarrow \mathbb{R}$, $\ell: \mathbb{R}^n \rightarrow \mathbb{R}^p$ and $\mathcal{W}: \mathbb{R}^n \rightarrow \mathbb{R}^{p \times m}$ such that V_s is C^1 and positive definite, $V_s(0) = 0$, and, for all $x \in \mathbb{R}^n$,

$$0 = V'_s(x)f(x) - h^T(x)Qh(x) + \ell^T(x)\ell(x) \tag{9}$$

$$0 = \frac{1}{2} V'_s(x)G(x) - h^T(x)(QJ(x) + S) + \ell^T(x)\mathcal{W}(x) \tag{10}$$

$$0 = R + S^T J(x) + J^T(x)S + J^T(x)QJ(x) - \mathcal{W}^T(x)\mathcal{W}(x) \tag{11}$$

The following special case is the nonlinear version of the Kalman–Yakubovich–Popov lemma.

Corollary 2.1

Let \mathcal{G} be zero-state observable and completely reachable. \mathcal{G} is passive if and only if there exist functions $V_s: \mathbb{R}^n \rightarrow \mathbb{R}$, $\ell: \mathbb{R}^n \rightarrow \mathbb{R}^p$, and $\mathcal{W}: \mathbb{R}^n \rightarrow \mathbb{R}^{p \times m}$ such that V_s is C^1 and positive definite, $V_s(0) = 0$, and for all $x \in \mathbb{R}^n$,

$$0 = V'_s(x)f(x) + \ell^T(x)\ell(x) \tag{12}$$

$$0 = \frac{1}{2} V'_s(x)G(x) - h^T(x) + \ell^T(x)\mathcal{W}(x) \tag{13}$$

$$0 = J(x) + J^T(x) - \mathcal{W}^T(x)\mathcal{W}(x) \tag{14}$$

Remark 2.2

Note that if (12)–(14) are satisfied and if, in addition, $\ell^T(x)\ell(x) > 0$ for all $x \neq 0$, then it follows that \mathcal{G} is state strict passive.

Remark 2.3

Using (12)–(14) it follows that

$$\int_{t_1}^{t_2} 2u^T(s)y(s) ds = V_s(x(t_2)) - V_s(x(t_1)) + \int_{t_1}^{t_2} [\ell^T(x(s)) + \mathcal{W}^T(x(s))u(s)]^T [\ell(x(s)) + \mathcal{W}(x(s))u(s)] ds \tag{15}$$

which can be interpreted as an energy balance equation where $V_s(x(t_2)) - V_s(x(t_1))$ is the stored or accumulated energy of the system and the second term on the right-hand side corresponds to the dissipated energy.¹¹

Next, we present a key result used later in the paper on linearization of passive systems.

Theorem 2.1

Consider the nonlinear plant \mathcal{G} given by (5) and (6) and assume that \mathcal{G} is passive. Then there exist matrices $P \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{p \times n}$ and $W \in \mathbb{R}^{m \times p}$ with P non-negative definite such that (1)–(3) are satisfied with

$$A = \left. \frac{\partial f}{\partial x} \right|_{x=0}, \quad B = G(0), \quad C = \left. \frac{\partial h}{\partial x} \right|_{x=0}, \quad D = J(0) \tag{16}$$

Proof. First note that since \mathcal{G} is passive there exist functions $V_s: \mathbb{R}^n \rightarrow \mathbb{R}$, $\ell: \mathbb{R}^n \rightarrow \mathbb{R}^p$, $\mathcal{W}: \mathbb{R}^n \rightarrow \mathbb{R}^{m \times p}$ with V_s non-negative definite, such that (12)–(14) are satisfied. Hence, there exists a non-negative-definite matrix $P \in \mathbb{R}^{n \times n}$ such that

$$V_s(x) = x^T P x + V_r(x)$$

where $V_r: \mathbb{R}^n \rightarrow \mathbb{R}$ contains the higher-order terms of $V_s(x)$. Next, let $f(x) = Ax + f_r(x)$, $\ell(x) = Lx + \ell_r(x)$, and $h(x) = Cx + h_r(x)$ where $f_r(x)$, $\ell_r(x)$, and $h_r(x)$ contain the nonlinear terms of $f(x)$, $\ell(x)$, and $h(x)$, respectively, and let $G(x) = B + G_r(x)$ and $\mathcal{W}(x) = W + \mathcal{W}_r(x)$ where $W \triangleq \mathcal{W}(0)$ and $G_r(x)$ and $\mathcal{W}_r(x)$ contain the non-constant terms of $G(x)$ and $\mathcal{W}(x)$, respectively. Using the above expressions (12) and (13) can be written as

$$0 = x^T (A^T P + PA + L^T L)x + \gamma(x) \tag{17}$$

$$0 = x^T (PB - C^T + L^T W) + \Gamma(x) \tag{18}$$

where

$$\begin{aligned} \gamma(x) &\triangleq V_r'(x) f(x) + \ell_r^T(x) \ell_r(x) + 2x^T (P f_r(x) + L^T \ell_r(x)) \\ \Gamma(x) &\triangleq \frac{1}{2} V_r'(x) G(x) - h_r^T(x) + x^T P G_r(x) + x^T L^T \mathcal{W}_r(x) + \ell_r^T(x) W \end{aligned}$$

Now, viewing (17) and (18) as the Taylor’s series expansion of (12) and (13), respectively, about $x = 0$ and noting that $\lim_{\|x\| \rightarrow 0} |\gamma(x)| / \|x\|^2 = 0$ and $\lim_{\|x\| \rightarrow 0} \|\Gamma(x)\| / \|x\|^2 = 0$ where $\|\cdot\|$ denotes the Euclidean vector norm, it follows that P satisfies (1) and (2). Finally, (3) follows from (14) by setting $x = 0$. □

In the case where \mathcal{G} is zero-state observable the storage function $V_s(x)$ in Theorem 2.1 is positive definite and, hence, the following corollary is immediate.

Corollary 2.2

Consider the nonlinear plant \mathcal{G} given by (5) and (6). Assume that \mathcal{G} is passive, zero-state observable, and exponentially stable and let A , B , C , and D be given by (16). Then there exist matrices $P \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{p \times n}$, and $W \in \mathbb{R}^{m \times p}$ with P positive definite such that (1)–(3) are satisfied. In this case

$$G(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

is positive real and asymptotically stable.

Proof. Since \mathcal{G} is zero-state observable and exponentially stable it follows that $V_s(x)$ given in the proof of Theorem (2.1) is positive definite and there exist positive constants α and β such that

$$\alpha \|x\|^2 \leq V_s(x) \leq \beta \|x\|^2, \quad x \in \mathbb{R}^n$$

Hence, there exists a positive-definite matrix $P \in \mathbb{R}^{n \times n}$ such that

$$V_s(x) = x^T P x + V_r(x)$$

where $V_r: \mathbb{R}^n \rightarrow \mathbb{R}$ contains higher-order terms of $V_s(x)$. Now (1)–(3) follow as in the proof of Theorem 2.1. The positive realness of $G(s)$ follows from Lemma 2.1 and Remark 2.1, while asymptotic stability follows from the fact that a nonlinear system is exponentially stable if and only if its linearization is asymptotically stable.²⁰ \square

3. NONLINEAR PASSIVE FEEDBACK SYSTEMS

In this section we provide a framework for designing nonlinear passive controllers for nonlinear passive systems. Specifically, consider the nonlinear plant \mathcal{G} given by (5) and (6) with the nonlinear feedback compensator \mathcal{G}_c given by

$$\dot{x}_c(t) = f_c(x_c(t)) + G_c(u_c(t), x_c(t)) u_c(t), \quad x_c(0) = x_{c0}, \quad t \geq 0 \tag{19}$$

$$y_c(t) = h_c(u_c(t), x_c(t)) + J_c(u_c(t), x_c(t)) u_c(t) \tag{20}$$

where $x_c \in \mathbb{R}^{n_c}$, $u_c, y_c \in \mathbb{R}^m$, $f_c: \mathbb{R}^{n_c} \rightarrow \mathbb{R}^{n_c}$, $G_c: \mathbb{R}^m \times \mathbb{R}^{n_c} \rightarrow \mathbb{R}^{n_c \times m}$, $h_c: \mathbb{R}^m \times \mathbb{R}^{n_c} \rightarrow \mathbb{R}^m$ and $J_c: \mathbb{R}^m \times \mathbb{R}^{n_c} \rightarrow \mathbb{R}^{m \times m}$. Note that with the feedback interconnection given by Figure 1, $u_c = y$ and $y_c = -u$. Furthermore, for generality, we allow the compensator to be of fixed dimension n_c that may be less than the plant order n . The following theorem gives sufficient conditions for global asymptotic stability of the feedback interconnection given by Figure 1.

Theorem 3.1 (Reference 10)

Consider the closed-loop system consisting of the nonlinear plant \mathcal{G} given by (5), (6) and the nonlinear compensator \mathcal{G}_c given by (19), (20). Let $\varepsilon, \hat{\varepsilon}, \varepsilon_c, \hat{\varepsilon}_c \in \mathbb{R}$ be such that $\varepsilon + \hat{\varepsilon} > 0$ and $\hat{\varepsilon} + \varepsilon_c > 0$, and assume \mathcal{G} and \mathcal{G}_c are zero-state observable. If \mathcal{G} is dissipative with respect to the supply rate $r(u, y) = u^T y - \varepsilon y^T y - \hat{\varepsilon} u^T u$ and \mathcal{G}_c is dissipative with respect to the supply rate $r(u_c, y_c) = u_c^T y_c - \varepsilon_c y_c^T y_c - \hat{\varepsilon}_c u_c^T u_c$, then the negative feedback interconnection of \mathcal{G} and \mathcal{G}_c is globally asymptotically stable.

Remark 3.1

Note that it follows from Theorem 3.1 that if \mathcal{G} and \mathcal{G}_c are input strict passive or output strict passive, then the negative feedback interconnection of \mathcal{G} and \mathcal{G}_c is asymptotically stable.

In order to obtain asymptotic stability in Theorem 3.1, we assumed that \mathcal{G} and \mathcal{G}_c are zero-state observable and input/output strict passive. The following theorem provides alternative conditions which guarantee asymptotic stability of the negative feedback interconnection of \mathcal{G} and \mathcal{G}_c when \mathcal{G} and \mathcal{G}_c are state strict passive.

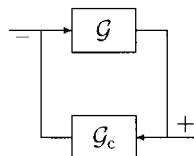


Fig. 1. Feedback interconnection of \mathcal{G} and \mathcal{G}_c

Theorem 3.2

Consider the closed-loop system consisting of the nonlinear plant \mathcal{G} given by (5), (6) and the nonlinear compensator \mathcal{G}_c given by (19), (20). Suppose there exist functions $V_s: \mathbb{R}^n \rightarrow \mathbb{R}$, $V_{sc}: \mathbb{R}^{n_c} \rightarrow \mathbb{R}$, $\ell: \mathbb{R}^n \rightarrow \mathbb{R}^p$, $\ell_c: \mathbb{R}^{n_c} \rightarrow \mathbb{R}^{p_c}$, $\mathcal{W}: \mathbb{R}^n \rightarrow \mathbb{R}^{p \times m}$, and $\mathcal{W}_c: \mathbb{R}^m \times \mathbb{R}^{n_c} \rightarrow \mathbb{R}^{p_c \times m}$, such that V_s and V_{sc} are C^1 and positive definite, $V_s(0) = 0$, $V_{sc}(0) = 0$, $V_s(x) \rightarrow \infty$ as $x \rightarrow \infty$ or $V_{sc}(x_c) \rightarrow \infty$ as $\|x_c\| \rightarrow \infty$, and

$$0 > V'_s(x) f(x) + \ell^T(x)\ell(x), \quad x \in \mathbb{R}^n, \quad x \neq 0 \tag{21}$$

$$0 = \frac{1}{2} V'_s(x)G(x) - h^T(x) + \ell^T(x)\mathcal{W}(x) \tag{22}$$

$$0 = J(x) + J^T(x) - \mathcal{W}^T(x)\mathcal{W}(x) \tag{23}$$

$$0 > V'_{sc}(x_c)f_c(x_c) + \ell_c^T(x_c)\ell_c(x_c), \quad x_c \in \mathbb{R}^{n_c}, \quad x_c \neq 0 \tag{24}$$

$$0 = \frac{1}{2} V'_{sc}(x_c)G_c(u_c, x_c) - h_c^T(u_c, x_c) + \ell_c^T(x_c)\mathcal{W}_c(u_c, x_c) \tag{25}$$

$$0 = J_c(u_c, x_c) + J_c^T(u_c, x_c) - \mathcal{W}_c^T(u_c, x_c)\mathcal{W}_c(u_c, x_c) \tag{26}$$

Then the negative feedback interconnection of \mathcal{G} and \mathcal{G}_c is globally asymptotically stable.

Proof. Consider the Lyapunov function candidate $V(x, x_c) = V_s(x) + V_{sc}(x_c)$. The corresponding Lyapunov derivative is given by

$$\dot{V}(x, x_c) = V'_s(x)(f(x) + G(x)u) + V'_{sc}(x_c)(f_c(x_c) + G_c(u_c, x_c)u_c)$$

which, using (21)–(26) and noting that $u_c = y$, implies

$$\begin{aligned} \dot{V}(x, x_c) &< -\ell^T(x)\ell(x) + 2h^T(x)u - 2\ell^T(x)\mathcal{W}(x)u - \ell_c^T(x_c)\ell_c(x_c) \\ &\quad + 2h_c^T(y, x_c)y - 2\ell_c^T(x_c)\mathcal{W}_c(y, x_c)y \\ &= 2h^T(x)u - [\ell(x) + \mathcal{W}(x)u]^T [\ell(x) + \mathcal{W}(x)u] + u^T (J(x) + J^T(x))u + 2h_c^T(y, x_c)y \\ &\quad - [\ell_c(x_c) + \mathcal{W}_c(y, x_c)y]^T [\ell_c(x_c) + \mathcal{W}_c(y, x_c)y] + y^T (J_c(y, x_c) + J_c^T(y, x_c))y \\ &= -[\ell(x) + \mathcal{W}(x)u]^T [\ell(x) + \mathcal{W}(x)u] - [\ell_c(x_c) + \mathcal{W}_c(y, x_c)y]^T [\ell_c(x_c) + \mathcal{W}_c(y, x_c)y] \\ &\leq 0 \end{aligned}$$

for all $(x, x_c) \neq (0, 0)$. Hence, since $V(x, x_c)$ is radially unbounded, the negative feedback interconnection of \mathcal{G} and \mathcal{G}_c is globally asymptotically stable. □

4. LINEAR POSITIVE REAL CONTROLLERS FOR PASSIVE SYSTEMS

In this section we begin by specializing the nonlinear passive feedback system framework of Section 3 to linear systems. Specifically, letting $f(x) = Ax$, $G(x) = B$, $h(x) = Cx$ and $J(x) = D$ yields

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \quad t \geq 0 \tag{27}$$

$$y(t) = Cx(t) + Du(t) \tag{28}$$

while $f_c(x_c) = A_c x_c$, $G_c(u_c, x_c) = B_c$, $h_c(u_c, x_c) = C_c x_c$ and $J_c(u_c, x_c) = D_c$ yields the linear dynamic compensator

$$\dot{x}_c(t) = A_c x_c(t) + B_c u_c(t), \quad x_c(0) = x_{c0} \tag{29}$$

$$y_c(t) = C_c x_c(t) + D_c u_c(t) \tag{30}$$

Although there is no general theory yet available for designing positive real controllers, an LQG-based technique is given in Reference 6 which shows that if the plant is positive real and the LQG design weights are chosen in a specified manner then the resulting LQG controller is guaranteed to be positive real. Next, since the nonlinear passive controller synthesis framework given in the next section is predicated on nonlinear modifications of linear positive real controllers, we extend the LQG-based positive real controller synthesis results of Reference 6. Specifically, the following theorem corresponds to a generalization of the results of Reference 6 since we do not require that the plant (27), (28) be positive real. Furthermore, our result accounts for a ‘cross-weighting’ term within the context of an LQG cost functional as well as for a direct transmission term in the plant dynamics.

Theorem 4.1

Assume

$$G(s) \underset{\sim}{\sim} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

is strictly positive real. Let $R_1 \in \mathbb{R}^{n \times n}$, $R_2 \in \mathbb{R}^{m \times m}$, and $R_{12} \in \mathbb{R}^{n \times m}$, with R_1 and R_2 positive definite, be such that

$$R_1 > R_{12} R_2^{-1} R_{12}^T + (1 + \alpha)^2 C^T (R_2 - (1 + \alpha)^2 (D + D^T))^{-1} C \tag{31}$$

$$R_2 > (1 + \alpha)^2 (D + D^T) \tag{32}$$

$$R_{12} = \alpha Y B \tag{33}$$

where $\alpha \in \mathbb{R}$ and $Y > 0$ satisfies

$$0 = A^T Y + Y A + R_1 - (B^T Y + R_{12}^T)^T R_2^{-1} (B^T Y + R_{12}^T) \tag{34}$$

Then the negative feedback dynamic compensator

$$G_c(s) \underset{\sim}{\sim} \left[\begin{array}{c|c} \frac{A - (1 + \alpha) B R_2^{-1} [C + B^T Y - (1 + \alpha) D R_2^{-1} B^T Y]}{(1 + \alpha) R_2^{-1} B^T Y} & (1 + \alpha) B R_2^{-1} \\ \hline & 0 \end{array} \right] \tag{35}$$

is strictly positive real and the closed-loop system (27)–(30) is globally asymptotically stable. Alternatively, if $D = 0$ and R_1 satisfies

$$R_1 = L^T L + (1 + \alpha)^2 C^T R_2^{-1} C \tag{36}$$

where L satisfies (1), then $R_{12} = \alpha C^T$. In this case the compensator (35) is given by

$$G_c(s) \underset{\sim}{\sim} \left[\begin{array}{c|c} \frac{A - 2(1 + \alpha) B R_2^{-1} C}{(1 + \alpha) R_2^{-1} C} & (1 + \alpha) B R_2^{-1} \\ \hline & 0 \end{array} \right] \tag{37}$$

Proof. First we show that $G_c(s)$ given by (35) is strictly positive real. Adding and subtracting $(1 + \alpha)BR_2^{-1}(C + B^T Y - (1 + \alpha)DR_2^{-1}B^T Y)$ to and from (34) yields

$$\begin{aligned} 0 &= [A - (1 + \alpha)BR_2^{-1}(C + B^T Y - (1 + \alpha)DR_2^{-1}B^T Y)]^T Y \\ &\quad + Y [A - (1 + \alpha)BR_2^{-1}(C + B^T Y - (1 + \alpha)DR_2^{-1}B^T Y)] \\ &\quad + R_1 - (B^T Y + R_{12}^T)^T R_2^{-1}(B^T Y + R_{12}^T) + (1 + \alpha)C^T R_2^{-1}B^T Y + (1 + \alpha)YBR_2^{-1}C \\ &\quad + (B^T Y + R_{12}^T)^T R_2^{-1}B^T Y + YBR_2^{-1}(B^T Y + R_{12}^T) - (1 + \alpha)^2 YBR_2^{-1}(D + D^T)B^T Y \end{aligned} \quad (38)$$

Using (35), (38) can be written as

$$\begin{aligned} -(A_c^T Y + YA_c) &= R_1 - (B^T Y + R_{12}^T)^T R_2^{-1}(B^T Y + R_{12}^T) \\ &\quad + (1 + \alpha)[(C + B^T Y + R_{12}^T)^T R_2^{-1}B^T Y + YBR_2^{-1}(C + B^T Y + R_{12}^T)] \\ &\quad - (1 + \alpha)^2 YBR_2^{-1}(D + D^T)B^T Y \\ &= R_1 - R_{12}R_2^{-1}R_{12}^T - (1 + \alpha)^2 C^T(R_2 - (1 + \alpha)^2(D + D^T))^{-1}C \\ &\quad + [(1 + \alpha)[(R_2 - (1 + \alpha)^2(D + D^T))^{-1}C + R_2^{-1}B^T Y]^T \\ &\quad \times (R_2 - (1 + \alpha)^2(D + D^T)) [(1 + \alpha)[(R_2 - (1 + \alpha)^2(D + D^T))^{-1}C \\ &\quad + R_2^{-1}B^T Y] \end{aligned} \quad (39)$$

or, equivalently, using (31)

$$A_c^T Y + YA_c = -L_c^T L_c < 0 \quad (40)$$

where L_c is the positive-definite square root of the positive-definite matrix on the right-hand side of (39). Furthermore, since $B_c = (1 + \alpha)BR_2^{-1}$, it follows that $B_c^T Y = C_c$ and hence $G_c(s)$ is strictly positive real. Consequently, asymptotic stability of the closed-loop system follows from Theorem 3.2 by noting that a strictly positive real linear system is state strict passive. Next, if $D = 0$ and R_1 satisfies (36), (34) can be written as

$$0 = A^T Y + YA + L^T L + (1 + \alpha)^2 C^T R_2^{-1}C - (B^T Y + R_{12}^T)^T R_2^{-1}(B^T Y + R_{12}^T) \quad (41)$$

Now, since $D = 0$, (3) implies that $W = 0$ and hence it follows from (1) and (2) that $Y = P$ is a solution to (41). Now, using (2) it follows that $R_{12} = \alpha YB = \alpha C^T$. Finally, substituting $R_{12} = \alpha C^T$ into (35) yields (37). \square

Remark 4.1

Inequality (31) assures that $A_c^T Y + YA_c < 0$. Note, however, that (31) cannot be verified *a priori* since it involves the matrix Y which satisfies (34). Alternatively, setting R_1 to (36) and $D = 0$ or setting $R_{12} = 0$ (i.e. setting $\alpha = 0$) yields *a priori* verifiable conditions on R_1 .

Remark 4.2

Note that the 'cross-weighting' term R_{12} and the plant direct transmission term D were not considered in Reference 6.

Remark 4.3

Setting $R_{12} = 0$, $D = 0$, and transforming the plant realization

$$G(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right]$$

to a self-dual basis,⁸ (35) reduces to the passive controller obtained in Reference 6.

Remark 4.4

Note that Theorem 4.1 also holds if $G(s)$ is positive real.⁶ Furthermore, it should be noted that the compensator given by (35) is strictly positive real, irrespective of whether or not $G(s)$ is positive real while the compensator given by (37) requires that $G(s)$ be positive real.

5. NONLINEAR PASSIVE CONTROLLER SYNTHESIS FOR PASSIVE SYSTEMS

In this section we extend the results of Section 4 to nonlinear full- and reduced-order passive controller design for nonlinear passive systems. Specifically, using Theorems 3.1 and 3.2, we present a family of globally asymptotically stabilizing nonlinear passive controllers. In particular, we introduce a nonlinear modification to the linear passive controller of Theorem 4.1 by including higher-order signals to the input and output of the controller dynamics while retaining passivity of the controller.

Theorem 5.1

Let $\varepsilon_c, \delta_c > 0$ be such that $\varepsilon_c \delta_c < 1$. Consider the closed-loop system consisting of the nonlinear plant \mathcal{G} given by (5), (6) and the nonlinear controller \mathcal{G}_c given by

$$\dot{x}_c(t) = [A_c - Y^{-1}M(u_c(t), x_c(t))]x_c(t) + (1 - 2\varepsilon_c \delta_c)[B_c + N(u_c(t), x_c(t))]u_c(t) \quad (42)$$

$$y_c(t) = [C_c + N^T(u_c(t), x_c(t))Y]x_c(t) + \delta_c u_c(t) \quad (43)$$

where A_c, B_c, C_c , and $Y > 0$ satisfy

$$0 > A_c^T Y + Y A_c \quad (44)$$

$$C_c = B_c^T Y \quad (45)$$

$N: \mathbb{R}^m \times \mathbb{R}^{n_c} \rightarrow \mathbb{R}^{n_c \times m}$, and

$$M(u_c, x_c) \triangleq \varepsilon_c [C_c + N^T(u_c, x_c)Y]^T [C_c + N^T(u_c, x_c)Y], \quad u_c \in \mathbb{R}^m, \quad x_c \in \mathbb{R}^{n_c} \quad (46)$$

If \mathcal{G} is passive and zero-state observable then the negative feedback interconnection of \mathcal{G} and \mathcal{G}_c is globally asymptotically stable.

Proof. Let $V_{sc}(x) = \frac{1}{2} x_c^T Y x_c$, $f_c(x_c) = [A_c - Y^{-1}M(u_c, x_c)]x_c$, $G_c(u_c, x_c) = (1 - 2\varepsilon_c \delta_c) [B_c + N(u_c, x_c)]$, and $h_c(u_c, x_c) = [C_c + N^T(u_c, x_c)Y]x_c$ so that $M(u_c, x_c) = \varepsilon_c h_c^T(u_c, x_c)h_c(u_c, x_c)$. In this case, for all $x_c \neq 0$,

$$V'_{sc}(x_c)f_c(x_c) + \varepsilon_c h_c^T(u_c, x_c)h_c(u_c, x_c) = \frac{1}{2} x_c^T (A_c^T Y + Y A_c)x_c < 0$$

and

$$\begin{aligned} \frac{1}{2} V'_{sc}(x)G_c(u_c, x_c) &= (\frac{1}{2} - \varepsilon_c \delta_c)x^T [YB_c + YN(u_c, x_c)] \\ &= (\frac{1}{2} - \varepsilon_c \delta_c)x^T [C_c + N^T(u_c, x_c)Y]^T \\ &= (\frac{1}{2} - \varepsilon_c \delta_c)h_c^T(u_c, x_c) \end{aligned}$$

Now, with $Q = -\varepsilon_c I$, $S = \frac{1}{2} I$, $R = -\hat{\varepsilon}_c I$, $\ell_c(x_c) = (1/\sqrt{2}) [-A_c^T Y - Y A_c]^{1/2} x_c$, $\mathcal{W}_c(x_c) \equiv 0$, $\hat{\varepsilon}_c = \delta_c(1 - \varepsilon_c \delta_c)$, and $J_c(u_c, x_c) \equiv 0$, it follows from Lemma 2.2 that \mathcal{G}_c is dissipative with respect to the supply rate $r(u_c, y_c) = u_c^T y_c - \varepsilon_c y_c^T y_c - \hat{\varepsilon}_c u_c^T u_c$. Furthermore, for all $x_c \neq 0$,

$$\dot{V}_{sc}(x_c) = V'_{sc}(x_c) [f_c(x_c) + G_c(u_c, x_c)u_c] < u_c^T y_c - \varepsilon_c y_c^T y_c - \hat{\varepsilon}_c u_c^T u_c$$

and, hence, \mathcal{G}_c is zero-state observable. The result now follows as a direct consequence of Theorem 3.1. □

Remark 5.1

Note that if \mathcal{G} is state strict passive in Theorem 5.1, then we can set $\delta_c = 0$ and take $M(u_c, x_c)$ to be an arbitrary non-negative-definite function. In this case global asymptotic stability of the negative feedback interconnection of \mathcal{G} and \mathcal{G}_c follows from Theorem 3.2.

Remark 5.2

The nonlinear passive controllers for single-input/single-output systems suggested in Reference 13 and further explored in Reference 14 to significantly enhance energy flow between the plant and compensator are a special case of the nonlinear passive controller parameterization given by (42) and (43). Specifically, setting

$$A_c = \begin{bmatrix} 0 & \Omega \\ -\Omega & -2\eta \end{bmatrix}, M(y, x_c) = 0, Y = I_n, C_c + N^T(y, x_c) = B_c^T + N^T(y, x_c) = [0 \quad \kappa y e^T]$$

where $\Omega = \text{diag}[\Omega_1, \dots, \Omega_n]$, $\Omega_i > 0$, $i = 1, \dots, n$, $\eta = \text{diag}[\eta_1, \dots, \eta_n]$, $\eta_i > 0$, $i = 1, \dots, n$, $e = [1, 1, \dots, 1]^T$, $\kappa > 0$, the controller proposed in Reference 13 is recovered. Alternatively, transforming

$$G(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right]$$

to a self-dual basis⁸ and setting $M(y, x_c) = S$, $Y = I_n$, $C_c + N^T(y, x_c) = B_c^T + N^T(y, x_c) = B_c^T y$, where S is an arbitrary skew-symmetric matrix and A_c, B_c, C_c , are given by (35), the controller-proposed in Reference 14 is recovered.

It is important to note that Theorem 5.1 yields nonlinear stabilizing controllers for zero-state observable passive plants so long as (A_c, B_c, C_c) is strictly positive real. In particular, fixed-order strictly positive real realizations for (A_c, B_c, C_c) can be constructed as driving-point impedances of passive electrical networks involving resistor-inductor (RL), resistor-capacitor (RC), resistor-inductor-capacitor (RLC), and inductor-capacitor (LC) combinations which exhibit interlacing pole-zero patterns on the negative real axis and ε -shifted imaginary axis,²² where $\varepsilon > 0$. In

this case Y in (42) and (43) satisfying (44) and (45) can easily be computed using standard LMI techniques.²³ Alternatively, using Theorem 2.1 and Corollary 2.2, it is always possible to guarantee that the linearization of a passive, zero-state observable, and exponentially stable nonlinear system is positive real. In this case (A_c, B_c, C_c) and Y satisfying (44) and (45) can be obtained using Theorem 4.1 wherein (A_c, B_c, C_c) serves as a positive real controller for the linearized passive plant. Furthermore, in this case, reduced-order strictly positive real compensators (A_c, B_c, C_c) can be obtained by reducing the linearized plant using standard balanced truncation techniques and then applying Theorems 4.1 and 5.1 to obtain a reduced-order nonlinear stabilizing controller (42), (43). Recall that (see Remark 4.4) the reduced-order model upon which the strictly positive real reduced-order controller (A_c, B_c, C_c) is predicated need not be positive real. Of course, a positive real reduced-order model can be obtained by using the balanced truncation technique given in Reference 24.

6. BENCHMARK NONLINEAR CONTROL DESIGN PROBLEM

In this section we apply the nonlinear dissipative controller synthesis framework developed in Section 5 to the rotational/translational proof-mass actuator (RTAC) nonlinear benchmark problem.¹⁵ The system (see Figure 2) involves an eccentric rotational inertia which acts as a proof-mass actuator mounted on a translational oscillator. The oscillator cart of mass M is connected to a fixed support via a linear spring of stiffness k . The cart is constrained to one-dimensional motion and the rotational proof-mass actuator consists of a mass m and mass moment of inertia I located a distance e from the center of mass of the cart. In Figure 2, N denotes the control torque applied to the proof mass. Since the motion is constrained to the horizontal plane the gravitational forces are not considered in the dynamic analysis.

Letting q, \dot{q}, θ , and $\dot{\theta}$ denote the translational position and velocity of the cart and the angular position and velocity of the rotational proof mass, respectively, the nonlinear dynamic equations of motion are given by

$$(M + m)\ddot{q} + kq = -me(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) \quad (47)$$

$$(I + me^2)\ddot{\theta} = -me\dot{q} \cos \theta + N \quad (48)$$

with problem data given in Table I.¹⁵

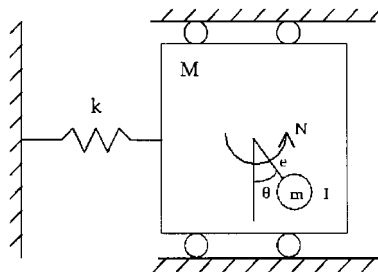


Fig. 2. Rotational/translational proof-mass actuator

Next, letting $x = [x_1 \ x_2 \ x_3 \ x_4]^T = [q \ \dot{q} \ \theta \ \dot{\theta}]^T$, the state-space form of (47) and (48) is given by

$$\dot{x} = f(x) + G(x)u \tag{49}$$

where

$$f(x) \triangleq \frac{1}{d(x)} \begin{bmatrix} d(x)x_2 \\ (I + me^2)(mex_4^2 \sin x_3 - kx_1) \\ d(x)x_4 \\ -me \cos x_3 (mex_4^2 \sin x_3 - kx_1) \end{bmatrix}, \quad G(x) \triangleq \frac{1}{d(x)} \begin{bmatrix} 0 \\ -me \cos x_3 \\ 0 \\ M + m \end{bmatrix} \tag{50}$$

$d(x) \triangleq M(I + me^2) + m(I + me^2 \sin^2 x_3)$, and $u \triangleq N$. Note that with output $y = x_4$ and input $u = N$, (49) is dissipative but not zero-state observable since $N \equiv 0, y \equiv 0$ does not imply $x \equiv 0$. Hence, we first design a control law $u = -k_\theta x_3 + \hat{u}$, where $k_\theta > 0$, that ensures that the closed-loop system is zero-state observable with output $y = x_4$ and input \hat{u} . In this case, (49) can be written as

$$\dot{x} = \hat{f}(x) + G(x)\hat{u} \tag{51}$$

where

$$\hat{f}(x) \triangleq \frac{1}{d(x)} \begin{bmatrix} d(x)x_2 \\ (I + me^2)(mex_4^2 \sin x_3 - kx_1) - me k_\theta x_3 \cos x_3 \\ d(x)x_4 \\ -me \cos x_3 (mex_4^2 \sin(x_3) - kx_1) - (M + m)k_\theta x_3 \end{bmatrix} \tag{52}$$

with associated positive-definite storage function given by

$$V_s(x) = \frac{1}{2} [kx_1^2 + (M + m)x_2^2 + k_\theta x_3^2 + (I + me^2)x_4^2 + 2mex_2 x_4 \cos x_3] \tag{53}$$

Table I. Problem data for the RTAC

Description	Parameter	Value	Units
Cart mass	M	1.3608	kg
Arm mass	m	0.096	kg
Arm eccentricity	e	0.0592	m
Arm inertia	I	0.0002175	kg m ²
Spring stiffness	k	186.3	N/m

Table II. Compensator structure and design variables

	Order	$N(y, x_c)$	R_1	R_2
Compensator 1	4	$B_c y$	I_4	10000
Compensator 2	2	0	I_2	1000
Compensator 3	2	$B_c y$	I_2	1000

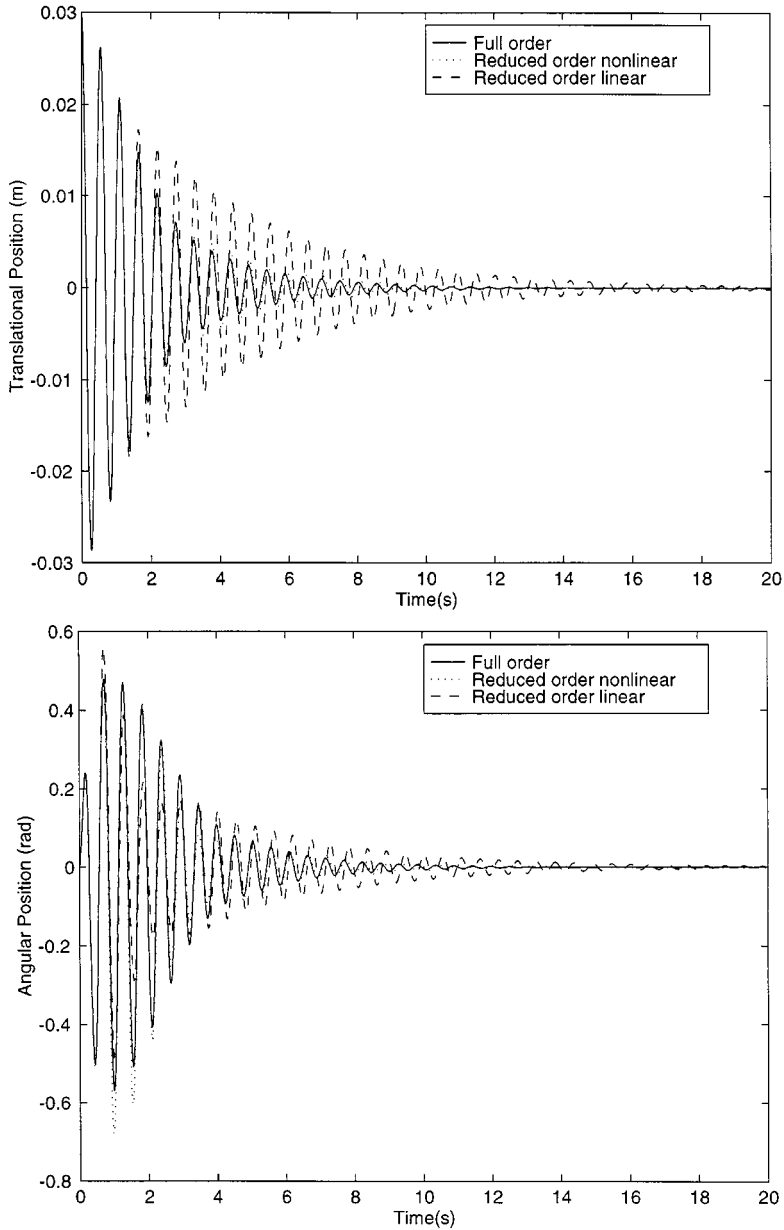


Fig. 3. Closed-loop response to 3 cm initial displacement

Using Theorems 2.1 and 5.1, full- and reduced-order globally asymptotically stabilizing linear and nonlinear dissipative controllers were designed for the case where $k_\theta = 0.057$, $\varepsilon_c = 0.5$, and $\delta_c = 0.00175$. For the reduced-order design, a linearized reduced-order model was obtained using the balanced reduction approach presented in Reference 24 and (A_c, B_c, C_c) and Y satisfying (42)

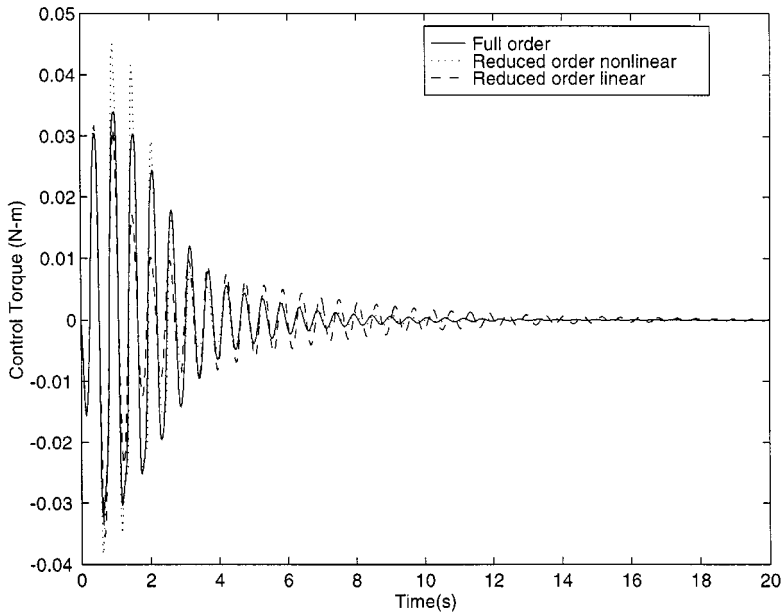


Fig. 4. Control torque

and (43) were obtained using Theorem 4.1. Table II lists the linear, $N(y, x_c) = 0$, and nonlinear, $N(y, x_c) = B_c y$, controller design variables and controller orders used. Specifically, the nonlinear fixed-order dynamic compensator considered is given by Theorem 5.1 with $N(u_c, x_c) = B_c y$ or

$$\dot{x}_c = [A_c - \varepsilon_c Y^{-1}(C_c + y^T B_c^T Y)^T (C_c + y^T B_c^T Y)]x_c + (1 - 2\varepsilon_c \delta_c)[B_c + B_c y]u_c \quad (54)$$

$$-u_c = (C_c + y^T B_c^T Y)x_c + \delta_c u_c \quad (55)$$

where (A_c, B_c, C_c) and Y were obtained using Theorem 4.1.

Figure 3 shows the controlled translational position response of the cart and controlled angular position response of the rotational proof mass for an initial position disturbance of 3 cm. Finally, Figure 4 shows the control torque for the different controllers. Note that the maximum torque is well within the specified limit of $|N| \leq 0.1 \text{ N m}$.¹⁵

7. CONCLUSIONS

Using nonlinear dissipation theory a method for designing full- and reduced-order dynamic output feedback passive controllers for passive systems was developed. In particular, the proposed approach extends existing linear positive real controller synthesis frameworks by providing a parameterization of nonlinear controllers that can serve to enhance system performance and energy dissipation. Finally, using the proposed framework, full- and reduced-order dynamic output feedback compensators were designed for the rotational/translational proof-mass actuator nonlinear benchmark problem.

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