

Robust Stabilization of Axial Flow Compressors with Uncertain Pressure-Flow Maps

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Abstract—In this paper we develop globally robustly stabilizing controllers for rotating stall and surge in multimode axial flow compressor models with uncertain pressure-flow compressor performance characteristic maps. Specifically, using Lyapunov stability theory, a novel switching nonlinear globally robustly stabilizing control law based on equilibria-dependent Lyapunov functions with converging domains of attraction is developed. The locus of the equilibrium points on which the equilibria-dependent Lyapunov functions are predicated is characterized by the axisymmetric stable pressure-flow equilibrium branch of the nominal compression system. The proposed robust nonlinear control framework is directly applicable to uncertain compression systems with actuator amplitude and rate saturation constraints while providing a guaranteed domain of attraction. An application to a two-mode axial flow compressor with an uncertain pressure-flow map and rate saturation control constraints is presented.

Index Terms—Amplitude and rate saturation, axial compressors, equilibria-dependent Lyapunov functions, parametric uncertainty, robust control, rotating stall, surge.

I. INTRODUCTION

THE desire for developing an integrated control-system design methodology for advanced propulsion systems has led to significant activity in modeling and control of axial flow compression systems in recent years (see, for example, [1]–[7] and the numerous references therein). However, unavoidable discrepancies between compression system models and real-world compression systems can result in degradation of control-system performance including instability. In particular, jet engine control systems predicated on uncertain compressor performance pressure-flow characteristic maps can severely limit jet engine compression system performance by inducing the compressor aerodynamics instabilities of rotating stall and surge [8]. Rotating stall is an inherently two-dimensional local compression system oscillation which is characterized by regions of flow that rotate at a fraction of the compressor rotor speed while surge is a one-dimensional axisymmetric global compression system oscillation which involves axial flow oscillations and in some cases even axial flow reversal which can damage engine components and cause flameout to occur.

In this paper we address the problem of nonlinear robust control for rotating stall and surge in axial flow compressors

with uncertain performance characteristic pressure-flow maps. As shown in [8], feedback controllers that do not account for the presence of uncertainty in the compressor-flow map can have adverse effects on compression system performance by driving the compression system to a stalled equilibrium or a surge limit cycle. Hence, it is of paramount importance that modeling pressure-flow map system uncertainty be accounted for in the control-system design process. System modeling errors such as uncertainty in the compressor performance pressure-flow characteristic, can be captured as structured parametric uncertainty. Parametric uncertainty refers to system errors that are modeled as real (possibly nonlinear) parameter uncertainties.

Using Lyapunov stability theory, the switching nonlinear globally stabilizing control framework for multimode axial flow compressor models based on equilibria-dependent Lyapunov functions with converging domains of attraction developed in [7], [9] is extended to account for pressure-flow map uncertainty in the compression system. The locus of equilibrium points on which the equilibria-dependent Lyapunov functions are predicated is characterized by the axisymmetric stable pressure-flow branch of the nominal compressor model. The proposed globally stabilizing robust switching control framework guarantees robust stability of a global attractor containing the maximum pressure operating point of the nominal performance characteristic pressure-flow map. The robust switching nonlinear controller is directly applicable to compression systems with actuator amplitude and rate saturation constraints while providing a guaranteed domain of attraction. Finally, we note that the proposed nonlinear robust stabilization framework provides the foundation to designing gain scheduled controllers for nonlinear systems and is not restricted to the multimode axial flow compressor model presented here. For further details see [9], [10].

II. GOVERNING FLUID DYNAMIC EQUATIONS FOR AXIAL COMPRESSION SYSTEMS

In this section we present a multimode model for capturing rotating stall and surge in axial flow compressors with uncertain pressure-flow maps. Specifically, we consider a basic compression system consisting of an inlet duct, a compressor, an outlet duct, a plenum, and a control throttle. We assume that the plenum dimensions are large as compared to the compressor-duct dimensions so that the fluid velocity and acceleration in the plenum are negligible. In this case, the pressure in the plenum is spatially uniform. Furthermore, we assume that the flow is controlled by a throttle at the plenum exit. Finally, we assume a low-speed compression system

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with oscillation frequencies much lower than the acoustic resonance frequencies so that the flow can be considered incompressible. However, we do assume that the gas in the plenum is compressible and hence acts as a gas spring. To capture post-stall transients in axial flow compression systems we use an n -mode Galerkin approximation model for the nonlinear partial differential equation characterizing the disturbance velocity potential at the compressor inlet proposed by Moore and Greitzer [3]. Specifically, invoking a momentum balance across the compression system, conservation of mass in the plenum, and using a Galerkin projection based on a n -mode circumferential spatial harmonic approximation for the nonaxisymmetric flow disturbances, yields

$$\frac{d\hat{\phi}}{d\xi} = A_S \hat{\phi} + D_S^{-1} \psi_C^{\text{nom}}(\hat{\phi}) - e\Psi, \quad \hat{\phi}(0) = \hat{\phi}_0 \quad (1)$$

$$\frac{d\Psi}{d\xi} = \frac{1}{4B^2 l_C} \left(\frac{e^T \hat{\phi}}{n_t} - \gamma \sqrt{\Psi} \right), \quad \Psi(0) = \Psi_0 \quad (2)$$

where $\hat{\phi} = [\phi_1 \ \phi_2 \ \dots \ \phi_{n_t}]^T$ is a vector of $n_t \triangleq 2n + 1$ axial flow coefficients measured around the compressor inlet annulus, Ψ is the normalized total-to-static pressure rise, ξ is a nondimensional time, γ is a parameter proportional to the control throttle opening, $e \triangleq [1 \ 1 \ \dots \ 1]^T \in \mathbb{R}^{n_t}$, l_C is the characteristic length of the compressor, B is a nondimensional compliance parameter, $\psi_C^{\text{nom}}(\hat{\phi}) \triangleq [\psi_C^{\text{nom}}(\phi_1) \ \psi_C^{\text{nom}}(\phi_2) \ \dots \ \psi_C^{\text{nom}}(\phi_{n_t})]^T \in \mathbb{R}^{n_t}$ is the nominal vector compressor characteristic map, and the system matrices $A_S, D_S \in \mathbb{R}^{n_t \times n_t}$ are functions of the compressor geometry and mode number. For complete details of the model see [7]. The compliance parameter B is a function of the compressor rotor speed and the system plenum size. For large values of B a surge limit cycle can occur while rotating stall can occur for any value of B . The vector compressor characteristic map elements are generally assumed to be cubic functions of the axial flow variables $\phi_i, i = 1, \dots, n_t$. In particular, the compressor characteristic considered in [3] is

$$\psi_C^{\text{nom}}(\phi_i) = \psi_{C_0} + H \left[1 + \frac{3}{2} \left(\frac{\phi_i}{W} - 1 \right) - \frac{1}{2} \left(\frac{\phi_i}{W} - 1 \right)^3 \right], \quad i = 1, \dots, n_t \quad (3)$$

where ψ_{C_0}, H , and W are parameters that can be used to shape the compressor characteristic map. In actual compressor data [6], [11] however, the compressor characteristic map exhibits a noncubic morphology that can drive the compression system to deep hysteresis during rotating stall. Hence, to account for compressor performance pressure-flow map uncertainties we assume that

$$\psi_C(\phi_i) \triangleq \psi_C^{\text{nom}}(\phi_i) + \delta\psi(\phi_i), \quad i = 1, \dots, n_t \quad (4)$$

where $\delta\psi(\phi_i), i = 1, \dots, n_t$, is an uncertain perturbation of the nominal compressor characteristic map

$\psi_C^{\text{nom}}(\phi_i), i = 1, \dots, n_t$. Hence, the new nonlinear uncertain state-space model becomes

$$\frac{d\hat{\phi}}{d\xi} = A_S \hat{\phi} + D_S^{-1} \left[\psi_C^{\text{nom}}(\hat{\phi}) + \Delta\psi_C(\hat{\phi}) \right] - e\Psi, \quad \hat{\phi}(0) = \hat{\phi}_0 \quad (5)$$

$$\frac{d\Psi}{d\xi} = \frac{1}{4B^2 l_C} \left(\frac{e^T \hat{\phi}}{n_t} - \gamma \sqrt{\Psi} \right), \quad \Psi(0) = \Psi_0 \quad (6)$$

where $\Delta\psi_C(\hat{\phi}) \triangleq [\delta\psi_C(\phi_1) \ \dots \ \delta\psi_C(\phi_{n_t})]^T$. Note that $(e^T \hat{\phi}/n_t) = \Phi$, where Φ is the circumferentially averaged axial mass flow at the compressor inlet, and, as shown in [7], A_S is skew symmetric and $D_S + D_S^T$ is positive definite. Furthermore, e is an eigenvector of A_S and D_S associated with the eigenvalues 0 and l_C , respectively. In the nominal case, i.e., $\Delta\psi_C(\hat{\phi}) \triangleq 0$, this state space representation is similar to that obtained in [11] using discrete Fourier transforms.

III. ROBUST STABILIZATION VIA EQUILIBRIA-DEPENDENT LYAPUNOV FUNCTIONS

In this section we develop a globally stabilizing robust control strategy for controlling the multimode uncertain compression system (5), (6). Specifically, using Lyapunov stability theory, a novel switching nonlinear globally stabilizing robust control law based on equilibria-dependent or, instantaneous (with respect to a given nominal equilibrium) Lyapunov functions, with converging domain of attractions is developed. The locus of equilibrium points on which the instantaneous Lyapunov functions are predicated is characterized by the axisymmetric stable pressure-flow equilibrium branch of the nominal system for a continuum of mass flow through the throttle. For this development define the shifted variables

$$\hat{\phi}_s \triangleq \frac{1}{W} (\hat{\phi} - 2W e), \quad \Psi_s \triangleq \frac{1}{H} (\Psi - \psi_{C_0} - 2H) \quad (7)$$

so that the maximum pressure point on the nominal compressor characteristic pressure-flow map is translated to the origin. In this case, the translated nonlinear uncertain system is given by

$$\begin{aligned} \dot{\hat{\phi}}_s &= A \hat{\phi}_s + P^{-1} \left[\psi_{sC}^{\text{nom}}(\hat{\phi}_s) + \Delta\psi_s(\hat{\phi}_s) \right] - e\Psi_s \\ \hat{\phi}_s(0) &= \hat{\phi}_{s0} \end{aligned} \quad (8)$$

$$\dot{\Psi}_s = \frac{1}{\beta^2} \left(\frac{e^T \hat{\phi}_s}{n_t} - u \right), \quad \Psi_s(0) = \Psi_{s0} \quad (9)$$

where

$$\begin{aligned} A &\triangleq \frac{W l_C}{H} A_S, \quad P \triangleq \frac{1}{l_C} D_S, \quad \beta \triangleq \frac{2BH}{W} \\ u &\triangleq \Phi_{sT} = \frac{\Phi_T}{W} - 2 \end{aligned} \quad (10)$$

$$\begin{aligned} \psi_{sC}^{\text{nom}}(\hat{\phi}_s) &\triangleq [\psi_{sC}^{\text{nom}}(\phi_{s1}) \ \dots \ \psi_{sC}^{\text{nom}}(\phi_{sn_t})]^T \\ \Delta\psi_s(\hat{\phi}_s) &\triangleq [\delta\psi_s(\phi_{s1}) \ \dots \ \delta\psi_s(\phi_{sn_t})]^T \end{aligned} \quad (11)$$

and $\dot{(\cdot)}$ represents differentiation with respect to nondimensional scaled time $t \triangleq (H/WL_C)\xi$.

Next, it follows from (4) that the actual compressor characteristic $\psi_{sC}(\phi_{si})$, $i = 1, \dots, n_t$, is given by

$$\psi_{sC}(\phi_{si}) = \psi_{sC}^{\text{nom}}(\phi_{si}) + \delta\psi_s(\phi_{si}), \quad i = 1, \dots, n_t \quad (12)$$

where $\psi_{sC}^{\text{nom}}(\phi_{si}) = -(3/2)\phi_{si}^2 - (1/2)\phi_{si}^3$ is the nominal compressor characteristic and $\delta\psi_s(\phi_{si})$, $i = 1, \dots, n_t$, is an uncertain perturbation of the nominal characteristic $\psi_{sC}^{\text{nom}}(\phi_{si})$, $i = 1, \dots, n_t$. Here, we assume

$$\delta\psi_s(\cdot) \in \Delta \triangleq \{\delta\psi_s: \mathbb{R} \rightarrow \mathbb{R}: [\delta\psi_s(y) - m_1(y)] \times [\delta\psi_s(y) - m_2(y)] \leq 0, y \in \mathbb{R}\} \quad (13)$$

where $m_1, m_2: \mathbb{R} \rightarrow \mathbb{R}$ are given arbitrary bounding functions. Now, note that for $u = \lambda$, where $\lambda \geq 0$, (8) and (9) with $\Delta\psi_s(\hat{\phi}_s) \equiv 0$ have an equilibrium point at $(\hat{\phi}_{s\lambda}, \Psi_{s\lambda})$, where $\hat{\phi}_{s\lambda} \triangleq \lambda e$ and $\Psi_{s\lambda} \triangleq \psi_{sC}^{\text{nom}}(\lambda) = -(3/2)\lambda^2 - (1/2)\lambda^3$.

A. Robust, Local Set Point Controller Designs

For the uncertain compression system (8) and (9), we now show that for $\lambda > 0$ there exists a robust control law such that a neighborhood \mathcal{N}_λ of the nominal equilibrium point $(\hat{\phi}_{s\lambda}, \Psi_{s\lambda})$ is locally asymptotically stable with domain of attraction \mathcal{D}_λ . To show this, consider the equilibrium-dependent Lyapunov function¹ candidate predicated on the nominal pressure-flow axisymmetric stable equilibria given by

$$V_\lambda(\hat{\phi}_s, \Psi_s) = \frac{1}{2n_t}(\hat{\phi}_s - \hat{\phi}_{s\lambda})^T P(\hat{\phi}_s - \hat{\phi}_{s\lambda}) + \frac{1}{2}\beta^2[\Psi_s - \Psi_{s\lambda}]^2 \quad (14)$$

with Lyapunov derivative

$$\dot{V}_\lambda(\hat{\phi}_s, \Psi_s) = -\frac{1}{n_t}(\hat{\phi}_s - \lambda e)^T [\psi_{sC}^{\text{nom}}(\lambda)e - \psi_{sC}^{\text{nom}}(\hat{\phi}_s) - \Delta\psi_s(\hat{\phi}_s)] - h_\lambda(\hat{\phi}_s, \Psi_s) [\Psi_s - \psi_{sC}^{\text{nom}}(\lambda)] \quad (15)$$

where $u(\hat{\phi}_s, \Psi_s) = u_\lambda(\hat{\phi}_s, \Psi_s) \triangleq \lambda + h_\lambda(\hat{\phi}_s, \Psi_s)$ and $h_\lambda: \mathbb{R}^{n_t} \times \mathbb{R} \rightarrow \mathbb{R}$ is such that $h_\lambda(\hat{\phi}_{s\lambda}, \Psi_{s\lambda}) = 0$. Now, requiring $\dot{V}_\lambda(\hat{\phi}_s, \Psi_s) < 0$, $(\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda \setminus \mathcal{N}_\lambda$, guarantees local robust stability of the attractor \mathcal{N}_λ . However, (15) is dependent on the system uncertainty and needs to be checked for all $\delta\psi_s(\phi_{si}) \in \Delta$, $i = 1, \dots, n_t$, and hence is unverifiable. To obtain verifiable conditions for robust stability we introduce an equilibrium-dependent bounding function $\Gamma_\lambda(\cdot)$ for the uncertainty set Δ such that $\Gamma_\lambda(\cdot)$ bounds Δ . Specifically, define $\Gamma_\lambda: \mathbb{R}^{n_t} \rightarrow \mathbb{R}$ by

$$\Gamma_\lambda(\hat{\phi}_s) \triangleq \frac{1}{4}[m_2(\hat{\phi}_s) - m_1(\hat{\phi}_s)]^T [m_2(\hat{\phi}_s) - m_1(\hat{\phi}_s)] + \frac{1}{4}(\hat{\phi}_s - \lambda e)^T (\hat{\phi}_s - \lambda e) + \frac{1}{2}(\hat{\phi}_s - \lambda e)^T [m_1(\hat{\phi}_s) + m_2(\hat{\phi}_s)] \quad (16)$$

¹Here Lyapunov stability and asymptotic stability are defined with respect to a compact set. Hence, Lyapunov stability theorem conditions are modified accordingly. See ([12, Th. 2.2, pp. 66–68],) for details.

where $m_1(\hat{\phi}_s) \triangleq [m_1(\phi_{s1}) \dots m_1(\phi_{sn_t})]^T$ and $m_2(\hat{\phi}_s) \triangleq [m_2(\phi_{s1}) \dots m_2(\phi_{sn_t})]^T$. Now, note that if $\delta\psi(\cdot) \in \Delta$ then

$$\begin{aligned} 0 &\leq \frac{1}{4}[m_1(\hat{\phi}_s) + m_2(\hat{\phi}_s) + \hat{\phi}_s - \lambda e - 2\Delta\psi_s(\hat{\phi}_s)]^T \\ &\quad \times [m_1(\hat{\phi}_s) + m_2(\hat{\phi}_s) + \hat{\phi}_s - \lambda e - 2\Delta\psi_s(\hat{\phi}_s)] \\ &\quad - [\Delta\psi_s(\hat{\phi}_s) - m_1(\hat{\phi}_s)]^T [\Delta\psi_s(\hat{\phi}_s) - m_2(\hat{\phi}_s)] \\ &= \frac{1}{4}[m_2(\hat{\phi}_s) - m_1(\hat{\phi}_s)]^T [m_2(\hat{\phi}_s) - m_1(\hat{\phi}_s)] \\ &\quad + \frac{1}{4}(\hat{\phi}_s - \lambda e)^T (\hat{\phi}_s - \lambda e) \\ &\quad + \frac{1}{2}(\hat{\phi}_s - \lambda e)^T [m_1(\hat{\phi}_s) + m_2(\hat{\phi}_s)] \\ &\quad - (\hat{\phi}_s - \lambda e)^T \Delta\psi_s(\hat{\phi}_s) \end{aligned}$$

and hence $(\hat{\phi}_s - \lambda e)^T \Delta\psi_s(\hat{\phi}_s) \leq \Gamma_\lambda(\hat{\phi}_s)$, $\delta\psi(\phi_{si}) \in \Delta$, $i = 1, \dots, n_t$. Now, requiring

$$\begin{aligned} &-\frac{1}{n_t}(\hat{\phi}_s - \lambda e)^T [\psi_{sC}^{\text{nom}}(\lambda)e - \psi_{sC}^{\text{nom}}(\hat{\phi}_s)] \\ &\quad - h_\lambda(\hat{\phi}_s, \Psi_s) [\Psi_s - \psi_{sC}^{\text{nom}}(\lambda)] \\ &\quad + \frac{1}{n_t}\Gamma_\lambda(\hat{\phi}_s) < 0, \quad (\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda \setminus \mathcal{N}_\lambda \quad (17) \end{aligned}$$

it follows from (15) that $\dot{V}_\lambda(\hat{\phi}_s, \Psi_s) < 0$, $(\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda \setminus \mathcal{N}_\lambda$.

Next, for simplicity of exposition we set $m_1(\cdot) = -m_2(\cdot) = m(\cdot)$, where $m: \mathbb{R} \rightarrow \mathbb{R}$ is a given arbitrary function. In this case, substituting the expression for $\psi_{sC}^{\text{nom}}(\hat{\phi}_s)$ into (17), it follows that

$$\begin{aligned} \dot{V}_\lambda(\hat{\phi}_s, \Psi_s) &\leq -\frac{1}{2n_t} \sum_{i=1}^{n_t} \left\{ (\phi_{si} - \lambda)^2 \left[(\phi_{si})^2 + (\lambda + 3)\phi_{si} \right. \right. \\ &\quad \left. \left. + \lambda(\lambda + 3) - \frac{1}{2} \right] - 2m^2(\phi_{si}) \right\} \\ &\quad - h_\lambda(\hat{\phi}_s, \Psi_s) [\Psi_s - \psi_{sC}^{\text{nom}}(\lambda)] < 0, \\ &\quad (\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda \setminus \mathcal{N}_\lambda. \quad (18) \end{aligned}$$

Now, a sufficient condition guaranteeing that $\dot{V}_\lambda(\hat{\phi}_s, \Psi_s) < 0$, $(\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda \setminus \mathcal{N}_\lambda$, is given by

$$\frac{1}{2n_t} \sum_{i=1}^{n_t} (\phi_{si} - \lambda)^2 p_{1\lambda}(\phi_{si}) > 0, \quad (\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda \quad (19)$$

$$\begin{aligned} &\frac{1}{2n_t} \sum_{i=1}^{n_t} (\phi_{si} - \lambda)^2 p_{2\lambda}(\phi_{si}) + h_\lambda(\hat{\phi}_s, \Psi_s) [\Psi_s - \psi_{sC}^{\text{nom}}(\lambda)] \\ &> \frac{1}{2n_t} \sum_{i=1}^{n_t} m^2(\phi_{si}), \quad (\hat{\phi}_s, \Psi_s) \notin \mathcal{N}_\lambda \quad (20) \end{aligned}$$

where $p_{1\lambda}(\phi_{si}) \triangleq a_{1\lambda}(\phi_{si})^2 + b_{1\lambda}\phi_{si} + c_{1\lambda}$ and $p_{2\lambda}(\phi_{si}) \triangleq a_{2\lambda}(\phi_{si})^2 + b_{2\lambda}\phi_{si} + c_{2\lambda}$ are such that

$$\begin{aligned} p_{1\lambda}(\phi_{si}) + p_{2\lambda}(\phi_{si}) &= (\phi_{si})^2 + (\lambda + 3)\phi_{si} \\ &\quad + \lambda(\lambda + 3) - \frac{1}{2}. \quad (21) \end{aligned}$$

Note that (19) is satisfied in a domain $\mathcal{D}_\lambda \neq \emptyset$ only if there exists $d_\lambda > 0$ such that $p_{1\lambda}(\phi_{si}) > 0$, $-d_\lambda < \phi_{si} - \lambda < d_\lambda$, $i = 1, \dots, n_t$, and, in order to satisfy (20), we require that $p_{2\lambda}(\phi_{si}) > 0$, $i = 1, \dots, n_t$. Hence, we require that $p_{1\lambda}(\lambda) > 0$

and $p_{2\lambda}(\lambda) > 0$. A particular choice of $h_\lambda(\cdot, \cdot)$ satisfying (20) is given by

$$h_\lambda(\hat{\phi}_s, \Psi_s) \triangleq w[\Psi_s - \psi_{sC}^{\text{nom}}(\lambda)]p(\hat{\phi}_s - \lambda e) \quad (22)$$

where $w: \mathbb{R} \rightarrow \mathbb{R}$ is such that $xw(x) > 0, x \neq 0$, and $p: \mathbb{R}^{n_t} \rightarrow \mathbb{R}$ is a positive-definite function. However, note that for $\hat{\phi}_s = \lambda e$ it is not possible to satisfy (20) and hence by continuity there exists a neighborhood of this point where (18) *cannot* be satisfied. Thus, we construct a robust control law such that a neighborhood \mathcal{N}_λ of the nominal equilibrium point $(\hat{\phi}_{s\lambda}, \Psi_{s\lambda})$ is robustly stabilized with a given domain of attraction.

Next, note that it follows from (21) that for all $0 < \lambda \leq \sqrt{7/6} - 1$, $p_{1\lambda}(\lambda) + p_{2\lambda}(\lambda) \leq 0$ and hence the necessary conditions $p_{1\lambda}(\lambda) > 0$ and $p_{2\lambda}(\lambda) > 0$ for satisfying (19) and (20) are violated. Furthermore, if $\lambda > \sqrt{14/3} - 1$, then $p_{1\lambda}(\phi_{si}) + p_{2\lambda}(\phi_{si}) > 0, i = 1, \dots, n_t$, which implies that it is always possible to choose $p_{1\lambda}(\cdot)$ such that $p_{1\lambda}(\phi_{si}) > 0, i = 1, \dots, n_t$. More generally, there exists $\lambda_0 \geq \sqrt{7/6} - 1$ and $\lambda_{\text{global}} > \sqrt{14/3} - 1$ such that \mathcal{D}_{λ_0} collapses to the equilibrium point and $\mathcal{D}_{\lambda_{\text{global}}}$ coincides with the whole state space. Note that λ_0 and λ_{global} are dependent on the particular choice of the coefficients $a_{1\lambda}, b_{1\lambda}, c_{1\lambda}, a_{2\lambda}, b_{2\lambda}$, and $c_{2\lambda}$.

Next, with $u(\hat{\phi}_s, \Psi_s) = u_\lambda(\hat{\phi}_s, \Psi_s)$, we provide an estimate of the domain of attraction \mathcal{D}_λ for (8), (9). In particular

$$\mathcal{D}_\lambda \triangleq \begin{cases} \{(\hat{\phi}_s, \Psi_s): V_\lambda(\hat{\phi}_s, \Psi_s) \leq k_{1\lambda}\}, & \lambda_0 \leq \lambda \leq \lambda_{\text{global}}, \\ \mathbb{R}^{n_t} \times \mathbb{R}, & \lambda > \lambda_{\text{global}} \end{cases} \quad (23)$$

$$\mathcal{N}_\lambda \triangleq \{(\hat{\phi}_s, \Psi_s): V_\lambda(\hat{\phi}_s, \Psi_s) \leq k_{2\lambda}\}, \quad \lambda \geq \lambda_0 \quad (24)$$

where

$$k_{1\lambda} \triangleq \frac{\mu}{2n_t} d_\lambda^2, \quad \mu \triangleq \left(\max_i \{P_{ii}^{-1}\} \right)^{-1} \quad (25)$$

and

$$k_{2\lambda} \triangleq \max_{(\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda} \frac{1}{2n_t} (\hat{\phi}_s - \hat{\phi}_{s\lambda})^T P (\hat{\phi}_s - \hat{\phi}_{s\lambda}) + \frac{1}{2} \beta^2 [\Psi_s - \Psi_{s\lambda}]^2 \quad (26)$$

subject to

$$\begin{aligned} & \frac{1}{2n_t} \sum_{i=1}^{n_t} (\phi_{si} - \lambda)^2 p_{2\lambda}(\phi_{si}) + h_\lambda(\hat{\phi}_s, \Psi_s) [\Psi_s - \psi_{sC}^{\text{nom}}(\lambda)] \\ & = \frac{1}{n_t} \sum_{i=1}^{n_t} m^2(\phi_{si}). \end{aligned} \quad (27)$$

The Lyapunov level surfaces $V_\lambda(\hat{\phi}_s, \Psi_s) = k_{1\lambda}$ and $V_\lambda(\hat{\phi}_s, \Psi_s) = k_{2\lambda}$ are defined such that the intersection of the boundary of \mathcal{D}_λ with the plane $\Psi_s = \Psi_{s\lambda}$ is a closed surface contained in the region $\{\hat{\phi}_s: -d_\lambda < \phi_{si} - \lambda < d_\lambda, i = 1, \dots, n_t\}$ and \mathcal{N}_λ contains the region where (20) is *not* satisfied, so that $V_\lambda(\hat{\phi}_s, \Psi_s) < 0$ for all $(\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda \setminus \mathcal{N}_\lambda$. Note that since $k_{1\lambda} \geq 0$ and $k_{2\lambda} \geq 0$ and $V_\lambda(\hat{\phi}_s, \Psi_s)$ is continuous and radially unbounded, \mathcal{N}_λ and \mathcal{D}_λ are compact sets for $\lambda \in [\lambda_0, \lambda_{\text{global}}]$, which further implies that \mathcal{N}_λ and \mathcal{D}_λ are positively invariant sets. Thus, if the state space trajectories of (8), (9) enter \mathcal{D}_λ , then \mathcal{N}_λ serves as an attractor. Now, to ensure that $\mathcal{N}_\lambda \subset \mathcal{D}_\lambda$ we require that $k_{1\lambda} > k_{2\lambda}$. A typical plot

for the level set values $k_{1\lambda}$ and $k_{2\lambda}$ as functions of λ is shown in Fig. 1. Note that there exists λ_{min} such that $k_{1\lambda_{\text{min}}} = k_{2\lambda_{\text{min}}}$ and hence $\mathcal{D}_{\lambda_{\text{min}}} = \mathcal{N}_{\lambda_{\text{min}}}$. Hence, requiring $\lambda > \lambda_{\text{min}}$ assures the necessary condition that $\mathcal{N}_\lambda \subset \mathcal{D}_\lambda$.

The coefficients of the two parabolas $p_{1\lambda}(\cdot)$ and $p_{2\lambda}(\cdot)$ must be such that (21) is satisfied along with the above stated necessary conditions. This leaves some degree of freedom in the choice of the coefficients $a_{1\lambda}, b_{1\lambda}, c_{1\lambda}, a_{2\lambda}, b_{2\lambda}$, and $c_{2\lambda}$, which can be used to maximize the domain of attraction \mathcal{D}_λ and minimize the attractor \mathcal{N}_λ . This leads to the following optimization problem for each λ :

$$\max_{a_{1\lambda}, b_{1\lambda}, c_{1\lambda}, a_{2\lambda}, b_{2\lambda}, c_{2\lambda}} \left(\lambda^2 - \frac{c_{1\lambda}}{a_{1\lambda}} \right) \quad (28)$$

subject to

$$a_{1\lambda} + a_{2\lambda} = 1 \quad (29)$$

$$b_{1\lambda} + b_{2\lambda} = \lambda + 3 \quad (30)$$

$$c_{1\lambda} + c_{2\lambda} = \lambda(\lambda + 3) - \frac{1}{2} \quad (31)$$

$$(q_\lambda - \lambda)^2 (a_{2\lambda} q_\lambda^2 + b_{2\lambda} q_\lambda + c_{2\lambda}) = 2n_t \kappa^2 \quad (32)$$

$$2a_{1\lambda} \lambda + b_{1\lambda} = 0 \quad (33)$$

$$a_{1\lambda} < 0 \quad (34)$$

$$b_{1\lambda}^2 - 4a_{1\lambda} c_{1\lambda} > 0 \quad (35)$$

$$b_{2\lambda}^2 - 4a_{2\lambda} c_{2\lambda} < 0 \quad (36)$$

where

$$q_\lambda \triangleq \frac{(2a_{2\lambda} \lambda - 3b_{2\lambda}) - \sqrt{(2a_{2\lambda} \lambda - 3b_{2\lambda})^2 - 16a_{2\lambda}(2c_{2\lambda} - b_{2\lambda})}}{8a_{2\lambda}}$$

and $m(\phi_{si}), i = 1, \dots, n_t$, is chosen to be a constant value $\kappa \in \mathbb{R}$. Note that, with the assumption that $p_{1\lambda}(\cdot)$ achieves a maximum at λ , the objective function given by (28) corresponds to maximizing d_λ^2 . Furthermore, conditions (29)–(31) are obtained by equating the coefficients of equal powers in (21). Condition (32) guarantees that $(\phi_{si} - \lambda)^2 p_{2\lambda}(\phi_{si}), i = 1, \dots, n_t$, is a convex function for all $\phi_{si}, i = 1, \dots, n_t$, so that \mathcal{N}_λ is minimized, while conditions (33)–(35) guarantee that $p_{1\lambda}(\cdot)$ achieves a maximum at λ and $p_{1\lambda}(\lambda) > 0$. Finally, (36) guarantees that $p_{2\lambda}(\cdot) > 0$.

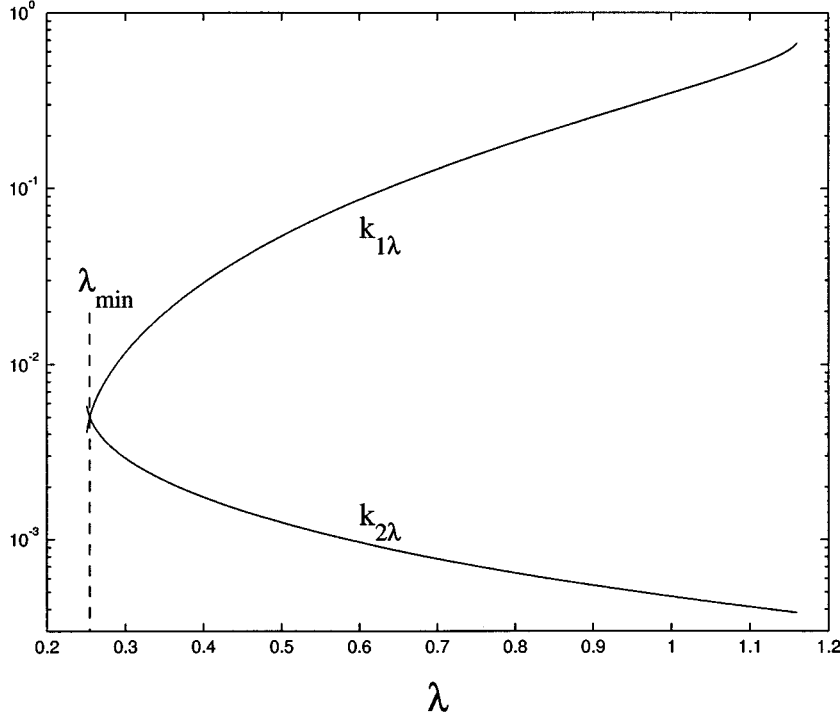


Fig. 1. Level set values $k_{1\lambda}$ and $k_{2\lambda}$ as functions of λ .

B. Global Robust Stabilization

In this subsection we present a globally robustly stabilizing control strategy for our compressor problem. For this development define the compact set

$$\mathcal{D} \triangleq \bigcup_{\lambda_{\min} \leq \lambda \leq \lambda_{\text{global}}} \mathcal{D}_\lambda \quad (37)$$

consisting of the union of the compact sets \mathcal{D}_λ , $\lambda \in [\lambda_{\min}, \lambda_{\text{global}}]$. Next, if $(\hat{\phi}_s(0), \Psi_s(0)) \notin \mathcal{D}$, setting $u(\hat{\phi}_s, \Psi_s) = u_{\hat{\lambda}}(\hat{\phi}_s, \Psi_s)$, $\hat{\lambda} \triangleq \lambda_{\text{global}} + \epsilon$, $\epsilon > 0$, the state trajectories $(\hat{\phi}_s(t), \Psi_s(t))$, $t \geq 0$, will approach the globally asymptotically stable invariant set $\mathcal{N}_{\hat{\lambda}}$ in a finite time. In particular, if ϵ is such that $\mathcal{N}_{\hat{\lambda}} \subseteq \mathcal{D}$ then there exists $t > 0$ such that $(\hat{\phi}_s(t), \Psi_s(t)) \in \mathcal{D}$. Now, let \bar{t} be such that $(\hat{\phi}_s(\bar{t}), \Psi_s(\bar{t})) \in \mathcal{D}$ and define

$$\lambda(\hat{\phi}_s, \Psi_s) \triangleq \inf_{\lambda > \lambda_{\min}} \{\lambda : (\hat{\phi}_s, \Psi_s) \in \mathcal{D}_\lambda\} \quad (38)$$

and $\bar{\lambda} \triangleq \lambda(\hat{\phi}_s(\bar{t}), \Psi_s(\bar{t}))$. From the definition of $\lambda(\cdot, \cdot)$ it follows that if $\bar{\lambda} > \lambda_{\min}$, then $(\hat{\phi}_s(\bar{t}), \Psi_s(\bar{t}))$ is on $\partial\mathcal{D}_{\bar{\lambda}}$, where $\partial\mathcal{D}_\lambda$ denotes the boundary of \mathcal{D}_λ , that is, $\partial\mathcal{D}_\lambda \triangleq \{(\hat{\phi}_s, \Psi_s) : V_\lambda(\hat{\phi}_s, \Psi_s) = k_{1\lambda}\}$. Furthermore, since $V_{\bar{\lambda}}(\hat{\phi}_s(\bar{t}), \Psi_s(\bar{t})) < 0$, $(\hat{\phi}_s, \Psi_s) \in \mathcal{D}_{\bar{\lambda}} \setminus \mathcal{N}_{\bar{\lambda}}$, it follows that there exists $\delta > 0$ such that $V_{\bar{\lambda}}(\hat{\phi}_s(t), \Psi_s(t)) < k_{1\bar{\lambda}}$, $t \in [\bar{t}, \bar{t} + \delta)$. Hence, $\lambda(\hat{\phi}_s(t), \Psi_s(t)) < \lambda(\hat{\phi}_s(\bar{t}), \Psi_s(\bar{t}))$, $t \in [\bar{t}, \bar{t} + \delta)$. Since \bar{t} was chosen arbitrarily, it follows that if $(\hat{\phi}_s(0), \Psi_s(0)) \in \mathcal{D} \setminus \mathcal{N}_{\lambda_{\min}}$ then $\lambda(\hat{\phi}_s(t), \Psi_s(t))$, $t \geq 0$, is monotonically decreasing.

Now, with $u(\hat{\phi}_s, \Psi_s) = u_{\lambda(\hat{\phi}_s, \Psi_s)}(\hat{\phi}_s, \Psi_s)$, where the notation $u_{\lambda(\hat{\phi}_s, \Psi_s)}(\hat{\phi}_s, \Psi_s)$ denotes a switching nonlinear feedback

controller with the switching function $\lambda(\hat{\phi}_s, \Psi_s)$ defined as in (38), consider the Lyapunov function candidate

$$\begin{aligned} V(\hat{\phi}_s, \Psi_s) &\triangleq V_{\lambda(\hat{\phi}_s, \Psi_s)}(\hat{\phi}_s, \Psi_s) - k_{1\lambda_{\min}} \\ &= k_{1\lambda(\hat{\phi}_s, \Psi_s)} - k_{1\lambda_{\min}} = \frac{\mu}{2nt} d_{\lambda(\hat{\phi}_s, \Psi_s)}^2 \\ &\quad - k_{1\lambda_{\min}}, \quad (\hat{\phi}_s, \Psi_s) \in \mathcal{D} \end{aligned} \quad (39)$$

with Lyapunov derivative

$$\dot{V}(\hat{\phi}_s, \Psi_s) = \frac{\mu}{nt} d_{\lambda(\hat{\phi}_s, \Psi_s)} \frac{d\lambda}{d\lambda}(\hat{\phi}_s, \Psi_s) \dot{\lambda}(\hat{\phi}_s, \Psi_s). \quad (40)$$

Since $\dot{\lambda}(\hat{\phi}_s, \Psi_s) < 0$ for $(\hat{\phi}_s, \Psi_s) \in \mathcal{D} \setminus \mathcal{N}_{\lambda_{\min}}$ and $(d\lambda/d\lambda) > 0$, it follows that $\dot{V}(\hat{\phi}_s, \Psi_s) < 0$ for $(\hat{\phi}_s, \Psi_s) \in \mathcal{D} \setminus \mathcal{N}_{\lambda_{\min}}$ and hence $\mathcal{N}_{\lambda_{\min}}$ is a locally asymptotically stable attractor. Now, to construct a globally stabilizing controller it need only to be noted that $u(\hat{\phi}_s, \Psi_s) = u_{\hat{\lambda}}(\hat{\phi}_s, \Psi_s)$ if $(\hat{\phi}_s(t), \Psi_s(t)) \notin \mathcal{D}$, $t \geq 0$, and $u(\hat{\phi}_s, \Psi_s) = u_{\lambda(\hat{\phi}_s, \Psi_s)}(\hat{\phi}_s, \Psi_s)$ otherwise, where $\lambda(\hat{\phi}_s, \Psi_s)$ is given by (38). However, this control law may be discontinuous at the boundary of \mathcal{D} . Alternatively, a continuous control law which globally stabilizes $\mathcal{N}_{\lambda_{\min}}$ can be obtained for $\epsilon \rightarrow 0^+$ and letting the state trajectories enter the domain $\mathcal{D}_{\lambda_{\text{global}}} \subset \mathcal{D}$ before switching the control law $u(\hat{\phi}_s, \Psi_s)$ from $u_{\hat{\lambda}}(\hat{\phi}_s, \Psi_s)$ to $u_{\lambda(\hat{\phi}_s, \Psi_s)}(\hat{\phi}_s, \Psi_s)$, where $\lambda(\hat{\phi}_s, \Psi_s)$ is given by (38).

The condition $V_\lambda(\hat{\phi}_s, \Psi_s) = k_{1\lambda}$ that characterizes the boundary of \mathcal{D}_λ , $\lambda \in [\lambda_{\min}, \lambda_{\text{global}}]$, depends on d_λ . However, the solution of this equation is not easily computable. An alternative approach for updating λ can be obtained by noting that the condition $V_\lambda(\hat{\phi}_s, \Psi_s) = k_{1\lambda}$ must be satisfied for all $t \geq 0$, and hence its time derivative must also be satisfied for all $t \geq 0$. In particular, using (40) and noting that

$\dot{V}(\hat{\phi}_s, \Psi_s) = \dot{V}_\lambda(\hat{\phi}_s, \Psi_s) + (\partial V_\lambda / \partial \lambda) \dot{\lambda}$, where $\dot{V}_\lambda(\hat{\phi}_s, \Psi_s)$ is given by (18), we obtain

$$\dot{\lambda} = \frac{\dot{V}_\lambda(\hat{\phi}_s, \Psi_s)}{\frac{\mu}{n_t} d_\lambda \frac{dd_\lambda}{d\lambda} - \lambda + \frac{e^T \hat{\phi}_s}{n_t} - 3\lambda\beta^2 (\Psi_s + \frac{3}{2}\lambda^2 + \frac{1}{2}\lambda^3) (1 + \frac{1}{2}\lambda)} \quad (41)$$

with $\lambda(0) = \lambda_0$ such that $V_{\lambda_0}(\hat{\phi}_{s_0}, \Psi_{s_0}) = k_{1\lambda_0}$. Note that (41) along with $u(\hat{\phi}_s, \Psi_s) = u_\lambda(\hat{\phi}_s, \Psi_s)$ provides a nonlinear first-order dynamic compensator equivalent to the original condition $V_\lambda(\hat{\phi}_s, \Psi_s) = k_{1\lambda}$ which now needs only to be solved once to compute the initial condition λ_0 . Also note that the compensator dynamics given by (41) characterize the admissible rate of the compensator state λ such that the switching nonlinear controller guarantees that $(\hat{\phi}_s(t), \Psi_s(t)) \in \partial \mathcal{D}_{\lambda(t)}$, $t \geq 0$.

Finally, since all control actuation devices are subject to amplitude and rate saturation constraints that lead to saturation nonlinearities, we discuss how the proposed switching nonlinear robust controller can be incorporated to address such practical limitations. Specifically, since the dynamic compensator state λ is proportional to the throttle opening (actuator) and since the dynamics given by (41) indirectly characterize the fastest admissible rate at which the control throttle can open while maintaining robust stability of the controlled system, it follows that by constraining the rate at which the dynamics of λ can evolve on the nominal equilibrium branch effectively places a rate constraint on the throttle opening. Mathematically, this corresponds to the case where the switching rate of the nonlinear robust controller is decreased so that the trajectory $(\hat{\phi}_s(t), \Psi_s(t))$, $t \geq 0$, is allowed to enter $\mathcal{D}_{\lambda(t)}$. Additionally, amplitude saturation constraints and state constraints can also be enforced by simply choosing $\lambda_{\max} < \lambda_{\text{global}}$ such that $\mathcal{D}_{\max} \triangleq \cup_{\lambda_{\min} \leq \lambda \leq \lambda_{\max}} \mathcal{D}_\lambda$ is contained in the region where the system is constrained to operate. In this case, the switching nonlinear robust controller provides a local stability guarantee with domain of attraction given by \mathcal{D}_{\max} . Of course, in practice it is sufficient to implement controllers with adequate domains of attraction and *a priori* saturation constraint guarantees rather than implementing global controllers without realistic actuator limitations.

IV. ROBUST CONTROL DESIGN FOR AN AXIAL FLOW COMPRESSOR

In this section we apply the globally stabilizing robust controller developed in Section III to a two mode axial flow compressor model with an uncertain compressor characteristic pressure-flow map. First, we consider the case where no actuator rate limitations are present on the throttle opening and then, to reflect a more realistic design, we assume that the throttle opening has a rate constraint of $|\dot{\gamma}| \leq 0.5$. Finally, we compare our results to the nonrobust equilibria-dependent nonlinear controller developed in [7].

Using the parameter values $a = 1/3$, $l_C = 6$, $m = 2$, $H = 0.32$, $W = 0.18$, $\psi_{C_0} = 0.23$, and $B = 0.1$, with initial conditions $\hat{\phi}_{s_0} = [0.700 \ -0.123 \ -0.776 \ -0.357 \ 0.556]^T$ and $\Psi_{s_0} = 0$ which, as shown in [7], correspond to a perturbation in the first-mode disturbance velocity potential, the proposed robustly globally stabilizing controller and the nonrobust equilibria-dependent controller developed in [7] were used to

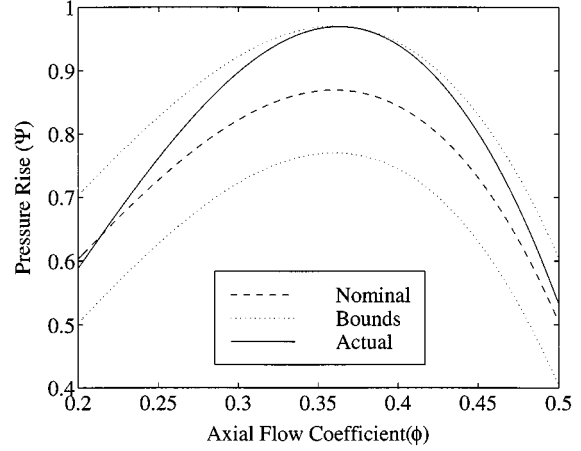


Fig. 2. Actual and nominal compressor characteristic maps.

compare the closed-loop system response. Here we model the uncertain perturbation to the nominal pressure-flow compressor characteristic map by

$$\delta\psi(\phi_{s_i}) = 0.1 \cos[10(\phi_{s_i} - 1)], \quad i = 1, \dots, 5. \quad (42)$$

Fig. 2 shows the nominal $\psi_C^{\text{nom}}(\hat{\phi})$ and actual $\psi_C(\hat{\phi})$ pressure-flow compressor characteristic maps for $\kappa = 0.1$. For this value of κ the optimization problem outlined in Section III for maximizing the domain of attraction \mathcal{D}_λ and minimizing the attractor \mathcal{N}_λ yields

$$\lambda_{\min} = 0.2547, \quad \lambda_{\text{global}} = 1.1604, \quad d_{\lambda_{\min}} = 0.2236 \\ k_{\lambda_{\min}} = 0.0050.$$

Finally, we use $u(\hat{\phi}_s, \Psi_s) = \lambda + h_\lambda(\hat{\phi}_s, \Psi_s)$, where $h_\lambda(\hat{\phi}_s, \Psi_s) = \Psi_s - \psi_C^{\text{nom}}(\lambda)$.

Fig. 3 shows the controlled responses for the squared stall cell amplitudes J_1 and J_2 which, as shown in [7], are explicitly related to ϕ_i , $i = 1, \dots, 5$, the circumferentially averaged compressor flow Φ , and the pressure rise Ψ for all three designs. This comparison illustrates that the robust controller globally stabilizes the axisymmetric operating point on the actual compressor characteristics corresponding to $(J_1, J_2, \Phi, \Psi) = (0, 0, 0.4133, 0.8471)$. Alternatively, the controller proposed in [7] drives the system to a limit-cycle instability induced by the control action. Note that the robust controller with the rate saturation constraint guarantees robust stability with minimal degradation in system performance. Finally, Fig. 4 shows the throttle opening and throttle rate versus time of the proposed robust controller, the proposed robust rate saturated controller, and the controller given in [7].

It is important to note that the control throttle is regulated through a switching feedback control strategy, with a switching function defined by (38), or, equivalently, (41). Alternatively, one can use the constant control $u = \lambda_{\text{global}}$ to attenuate non-axisymmetric disturbances and stabilize an operating condition on the pressure-flow map that is *far* from the maximum pressure operating point. However, this is not satisfactory since the design goal is to stabilize a point close to the maximum pressure operating point. In contrast, the proposed feedback control strategy provides robust stability and performance while driving

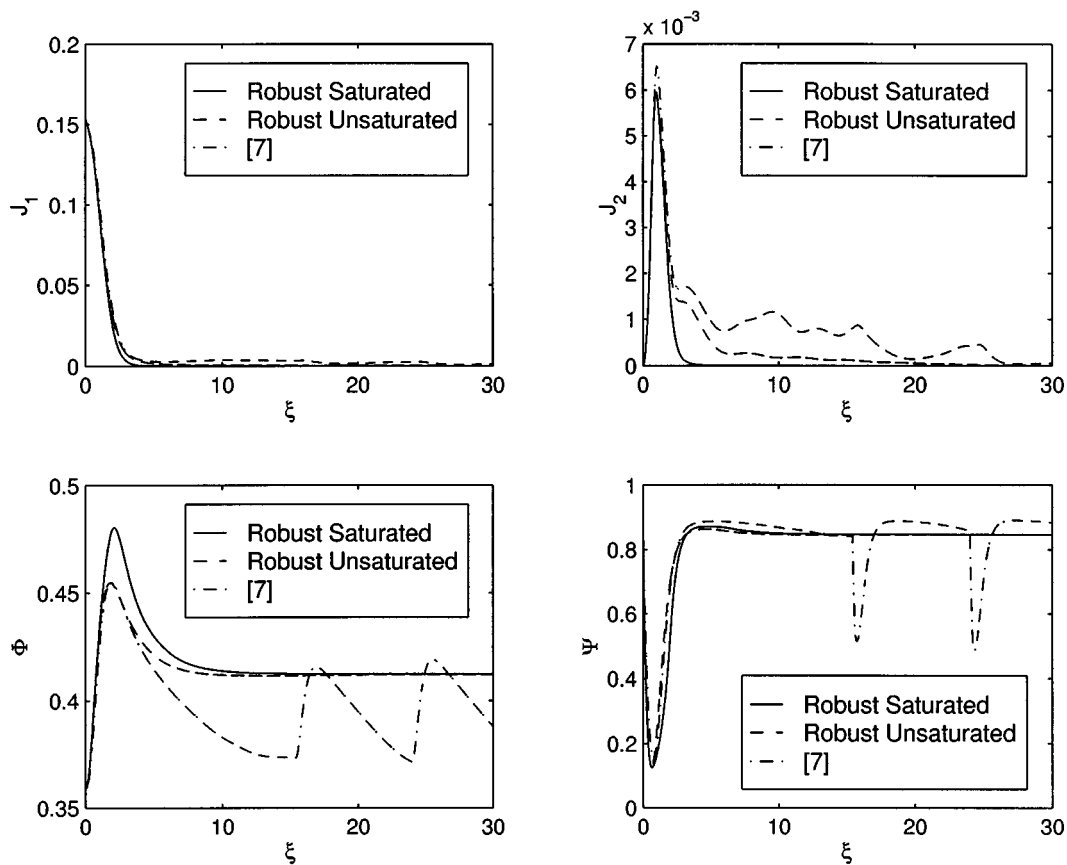


Fig. 3. Controlled squared stall amplitudes, flow, and pressure versus time.

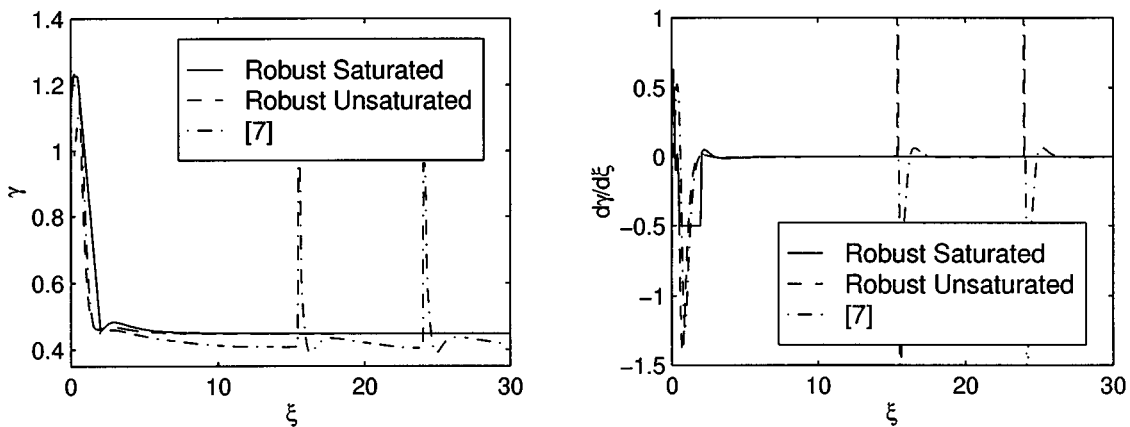


Fig. 4. Throttle opening versus time.

the controlled trajectory close to the maximum pressure operating point.

V. CONCLUSION

Using Lyapunov stability theory, a nonlinear globally robustly stabilizing control law based on equilibria-dependent Lyapunov functions with converging domains of attraction was developed for controlling rotating stall and surge in axial flow compressors. The proposed switching nonlinear robust control framework was shown to be directly applicable to compression systems with an uncertain pressure-flow compressor perfor-

mance characteristic maps and actuator amplitude and rate saturation constraints while providing a guaranteed domain of attraction to an attractor.

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