Time Delay and Noise Coupling in Limiting Control Effectiveness in Unstable Combustors

Jack H. Crawford, III1 and Tim C. Lieuwen2

Georgia Institute of Technology, Atlanta, GA, 30332

Combustion instabilities are a significant problem encountered in low NOx, premixed combustors used for ground based or aero-applications. Current capabilities to predict the conditions under which instabilities occur are immature, often resulting in the appearance of expensive and problematic instabilities late in the development stage of an engine. Consequently, there is significant interest in development of active instability control systems that can suppress an unexpected instability. While significant progress has been made in demonstrating that active instability control is feasible, the results of control implementation are highly variable. Much work remains in understanding the factors influencing how effective control will be for a given system. This paper describes a theoretical analysis of the statistics of a self excited combustor with closed loop feedback control. In particular, influence of the controller upon regions over which the system can be stabilized is investigated, as well as the limit cycle amplitude and degree of “amplitude breathing” of the controlled combustor. Additionally, the sensitivity of these control effectiveness figures of merit to variations in controller characteristics is explored. The model predictions are consistent with the experimental observation that the same control system can have dramatically different effects upon the controlled combustor amplitude. For example, this was demonstrated by showing that the effect of the same controller is quite different for different unstable combustors with different internal time delays – this can include the controller having essentially no useful effect at all.

Nomenclature

\[ a = \text{amplitude} \]
\[ c = \text{speed of sound} \]
\[ c_p = \text{specific heat at constant pressure} \]
\[ c_v = \text{coefficient of variation} \]
\[ DDE = \text{delay differential equation} \]
\[ \varepsilon = \text{heat release gain} \]
\[ E = \text{incomplete elliptical integral of the second kind} \]
\[ f = \text{pertaining to the flame} \]
\[ \eta = \text{temporal mode shape} \]
\[ \gamma = \text{ratio of specific heats} \]
\[ j = \sqrt{(-1)} \]
\[ J = \text{Jacobian} \]
\[ L = \text{combustor length} \]
\[ L_f = \text{flame length} \]
\[ LQG = \text{linear quadratic Gaussian} \]
\[ LTR = \text{loop transfer recovery} \]
\[ M = \text{Mach number} \]
\[ M\text{SEK} = \text{multi-scale extended Kalman} \]
\[ \mu = \text{mean state vector} \]
\[ n = \text{gain} \]
\[ \nu = \text{spectral frequency} \]
\[ \omega = \text{angular frequency} \]
\[ PDF = \text{probability density function} \]
\[ \phi = \text{equivalence ratio} \]
\[ \phi = \text{phase} \]
\[ \Phi = \text{power spectrum} \]
\[ \psi = \text{spatial mode shape} \]
\[ Q = \text{volumetric heat release rate} \]
\[ q = \text{heat flux rate} \]
\[ r = \text{amplitude} \]
\[ \rho = \text{density} \]
\[ SDDE = \text{stochastic delay differential equation} \]
\[ \sigma = \text{standard deviation} \]
\[ \Sigma = \text{state covariance matrix} \]
\[ St = \text{Strouhal number} \]
\[ SPL = \text{sound power level} \]
\[ STR = \text{self-tuning regulator} \]

1 Graduate Research Assistant, School of Aerospace Engineering, 270 Ferst Drive, Student Member
2 Associate Professor, School of Aerospace Engineering, 270 Ferst Drive, Lifetime Associate Fellow

Copyright © 2010 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.
Simultaneously, a higher frequency combustion instability can be excited from the effect of anti-resonance
constructively add if paired incorrectly with the phase roll-off region of the flame transfer function.
Between the stop-band and the pass-band of the band-pass filter is a region of rapid phase variation that can
excitation of a lower frequency instability compared to the nominally unstable frequency. In the frequency region
example is feeding the band-pass filtered output of the flame transfer function into the controller, leading to
frequency by only concentrating on oscillations at the nominally unstable frequency can lead to peak splitting. An
acoustic mode coupling.

Combustion instabilities are a common problem encountered in the development of combustion systems in power
generation and propulsion applications. Self-excited, combustion driven, oscillations often occur near natural
acoustic frequencies of the combustion system. Current capabilities to predict the conditions under which
instabilities occur are immature, often resulting in the appearance of expensive and problematic instabilities late in
the development stage of an engine. Moreover, designing systems that are stable over the range of required
operating conditions is very challenging. For this reason, there is significant interest in development of active
instability control systems that can suppress an unexpected instability that appears during testing or to broaden
operability ranges.

Suppression of combustion instabilities by feedback control has shown great potential over the last two decades.
Multiple control schemes have been used successfully with a wide selection of sensors, actuators, and combustor
geometries. Experiments often show significant suppression of combustion instabilities accompanied by an overall
lowering of the SPL over the entire frequency spectrum. In fact, an active control system has been successfully
commercialized and fielded by Siemens for control of azimuthal instabilities in their V94.3A heavy duty gas
turbines. To illustrate the range of active control demonstrations, Table 2 in the appendix summarizes experimental
results from a number of equivalence ratio modulation studies, showing the range of frequencies, fuels, and
operating conditions for which active control has been applied.

While significant progress has been made in demonstrating that active instability control is feasible, much work
remains in understanding the factors influencing how effective control will be for a given system. The results of
control implementation are highly variable. For example, even with their success, Siemens reported operating
conditions where feedback control would excite combustion instabilities at other frequencies. In many applications,
active control fails to fully suppress the original instability. Reported suppression values summarized in Table 2
show the very wide range in effects that active control implementation can have.

The example of Siemens’ experience with active control limitations is not atypical; new combustion dynamics
induced by feedback control is well documented in compilations in the combustion instability literature. A variety
of factors can lead to compromised capabilities to suppress combustion instabilities. These include background noise
floor levels, actuator control authority, actuator bandwidth, sensor location, and controlled combustor dynamics.
Under the influence of feedback control, the limit cycle pressure response can exhibit a variety of behaviors
associated with limited control effectiveness, such as peak splitting and amplitude breathing.

Peak splitting manifests itself as a growth in oscillation amplitude at closely spaced frequencies above and below
the nominal instability frequency. An example of peak splitting due to control is illustrated in Figure 1, which
contrasts the SPL of an adaptive controller from Kopasakis and Delaat with the uncontrolled case. Generally,
increased control authority attenuates the oscillation amplitude at the nominal instability frequency but sideband
frequencies intensify and move away from each other in the frequency domain. This peak splitting behavior has
been observed across a wide selection of control designs such as a phase shifter by Cohen et al., a LQG by
Murugappan et al., fuzzy logic control by Coker et al., and an adaptive sliding phase controller by Kopasakis et
al. The overall effect of peak splitting is to limit the maximum degree to which the oscillation amplitude can be
suppressed.

Investigations into the causes of peak splitting have focused on the discrepancy between modeled control
systems and their realizations. Fleifil et al. looked at the phase of the closed loop transfer function between the
combustor, controller, and filter dynamics using acoustic feedback. This analysis built upon earlier work on linear
acoustic mode coupling. They noted that ignoring phase effects across a frequency range around the instability
frequency by only concentrating on oscillations at the nominally unstable frequency can lead to peak splitting. An
example is feeding the band-pass filtered output of the flame transfer function into the controller, leading to
excitation of a lower frequency instability compared to the nominally unstable frequency. In the frequency region
between the stop-band and the pass-band of the band-pass filter is a region of rapid phase variation that can
constructively add if paired incorrectly with the phase roll-off region of the flame transfer function.
Simultaneously, a higher frequency combustion instability can be excited from the effect of anti-resonance

\[ \theta = \text{total phase} \quad W = \text{pertaining to noise} \]

\[ t = \text{time} \quad X = \text{auxiliary random variable} \]

\[ u = \text{velocity} \quad Y = \text{auxiliary random variable} \]

\[ \tau = \text{time delay} \quad \zeta = \text{damping coefficient} \]

\[ \xi = \text{unit variance Gaussian white noise} \]
interacting with a phase shifting controller. The system transfer function has zeros in it due to linear modal coupling, which creates a dip in the pressure response. In frequencies adjacent to the system zero is a region of rapid phase roll-off which can be excited if paired incorrectly with a phase shifting controller. The combination of the suppression of the nominally unstable frequency and the excitation of two adjacent frequencies results in the observation of peak splitting.

Further studies of peak splitting came from Banaszuk et al.\textsuperscript{15} using sensitivity function\textsuperscript{16} analysis to investigate combustors forced by background noise with time delayed control and self-excitation. Both the controller and the self-excitation were assumed to be governed by the same time delay. The sensitivity function used to analyze the effect of control is defined as the transfer function between output disturbances and the system output. Amplification or attenuation of a given frequency is dictated by the magnitude of the sensitivity function. Finite bandwidth control actuators (such as on-off valves) are limited over the range of frequencies that they can manipulate in the sensitivity function. As the magnitude of the sensitivity function is attenuated at the instability frequency, it must be accompanied by a magnification of adjacent frequencies, because the area of the logarithm of the sensitivity function is constant for all open loop stable controllers. This amplification of adjacent frequencies manifests itself as peak splitting. The presence of time delays is not needed for peak splitting, but their presence aggravates the problem. Simulations correctly reproduced peak splitting frequencies and amplitudes seen in experimental results.

Significant breathing in instability amplitude is another typical observation in actively controlled combustors.\textsuperscript{17} In this case, the average instability amplitude of the controlled combustor might be reduced significantly, but is associated with significant variations in instability amplitude on a cycle to cycle basis. This is a serious issue given that one of these significant amplitude bursts could result in blowoff of the flame, among other problems. Typical data illustrating this breathing phenomenon was reported by Johnson et al.\textsuperscript{2} and reproduced in Figure 2 above. Also shown on this plot is a sinusoidal signal with fixed phase. By comparing the phases of the pressure and this fixed phase signal it can be seen that there are time intervals of rapid phase drift in a combustor at low instability amplitude values. This can be seen at $t = 0.265$ and $0.285$ seconds, where the phase of the pressure signal abruptly changes by almost 180 degrees relative to the phase locked reference signal. The instability then grows in amplitude until the observer can converge to the correct phase. These data suggest that phase drift, coupled with observer dynamics at low signal to noise ratios of the coherent signal, lead to this phenomenon. In a related study, Lieuwen\textsuperscript{18} similarly found stochastic behavior in the phase of the uncontrolled combustor dynamics, where the phase drift per cycle was found to resemble a random walk.

The objective of this paper is to further consider the factors limiting the effectiveness of an active control system. In particular, focus is placed upon the influence of nominal combustor dynamics (i.e., the dynamics of the combustor system without control) upon the performance of an active control system. It will be shown that the same control system, even with a perfect observer, can lead to significantly different limit cycle pressure amplitudes of the controlled combustor. Specific focus is placed upon self-excited combustors in the presence of background noise, with independent internal and control-induced time delays, complementing the work of Banaszuk et al.\textsuperscript{15} The presence of multiple time delays and random noise introduces significant analytical complexities into the problem, but they are critical for describing the important combustion dynamics. Section II describes the modeling of

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{Figure1.png}
\caption{SPL of uncontrolled (solid) vs. controlled (dash) combustor pressure. Reproduced from Kopasakis & Delaat.\textsuperscript{1}}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{Figure2.png}
\caption{Comparison of the time variation of pressure in a controlled combustor and a phased locked reference. Reproduced from Johnson et al.\textsuperscript{2}}
\end{figure}
combustion instabilities, using a proportional time delayed controller with an ideal observer. This leads to a system of stochastic delay differential equations (SDDE) describing the temporal evolution of the acoustic pressure. Section III uses statistical and spectral analyses to solve for the probability density function (PDF) of the pressure amplitude and phase. The resulting expressions explicitly quantify the first and second moments of the limit cycle amplitude, with and without control. Using these expressions, Section IV presents and discusses stability and performance maps in a parameter space consisting of the gains and time delays between competing heat release source terms. Trends are analyzed for their impact in limiting control effectiveness. Implications of this paper on future research are discussed in section V.

II. Thermo-Acoustic Modeling

Development of a system of SDDE’s in this paper closely follows Culick’s derivation\(^1\) of equations of unsteady motion in a combustor.\(^2\) The reader is referred to his publications for the proof of the final linearized wave equation given below. Key assumptions in this equation are perfect gas behavior, low Mach number flow, and small perturbation amplitudes. Consequently, terms second order and greater in perturbation amplitude are neglected. As such, this equation cannot be used to capture the combustor dynamics under unstable conditions where system nonlinearities are needed.

\[
\frac{\partial^2 p'}{\partial t^2} - c^2 \nabla^2 p' = \left[ \gamma \rho \nabla \cdot (\bar{u} \cdot \nabla u') + \gamma \rho \nabla \cdot (u' \cdot \nabla \bar{u}) - \gamma \frac{\partial p'}{\partial t} \nabla \cdot \bar{u} - \bar{u} \cdot \nabla \frac{\partial p'}{\partial t} \right] + (\gamma - 1) \frac{\partial Q'}{\partial t} \tag{1}
\]

Each perturbed term in Eq. (1) can be subdivided into three components: acoustic, entropic, and vorticity.\(^2\) Any term not purely acoustic in nature is treated here as a random background noise source. These noise sources are both additive and multiplicative in nature, and describe such processes as heat release fluctuations due to turbulence or acoustic wave scattering by random temperature and velocity fluctuations. Currently, there is a lack of realistic physical descriptions of these noise processes, such as their amplitudes and correlations. Fairly rigorous expressions for the noise expressions due to gas dynamic processes have been presented by Burnley.\(^2\) Stochastic heat release terms are probably much more significant, but have not been developed from rigorous analyses; we are only aware of heuristic estimates of their characteristics.\(^2\) For analytic simplicity, we only include additive noise terms in this analysis. Note, however, when the nonlinearity introduced by multiplicative noise is sufficiently weak, approximate statistical or probabilistic linearization techniques can be used to recover equations with modified coefficients that behave as if driven solely by additive noise.\(^2\)

We next consider the unsteady heat release model. Two key assumptions invoked here is that the flame is acoustically compact and that the contributions of the self-excited combustion instabilities and the active control to the unsteady heat release, \(Q_{self}\) and \(Q_{cont}\), are decoupled and can be combined additively. The first assumption allows for the treatment of the heat release as concentrated at a point, because the flame appears as a discontinuity to a sound wave. Heat release decoupling allows the additive decomposition illustrated in Eq. (2).

\[
\frac{\partial Q(\tilde{x}, \tilde{t})}{\partial \tilde{t}} = \frac{\partial Q_{self}(\tilde{x}, \tilde{t})}{\partial \tilde{t}} + \frac{\partial Q_{cont}(\tilde{x}, \tilde{t})}{\partial \tilde{t}} \tag{2}
\]

We assume in this paper that the self-excitation is due to velocity fluctuations. Control is assumed to be implemented by secondary fuel injection using pressure based feedback.

\[
\frac{\partial Q(\tilde{x}, \tilde{t})}{\partial \tilde{t}} = \frac{\partial Q_{v}(\tilde{x}, \tilde{t})}{\partial \tilde{t}} + \frac{\partial Q_{\phi}(\tilde{x}, \tilde{t})}{\partial \tilde{t}} \tag{3}
\]

General expressions for the relationship between velocity and fuel/air ratio perturbations, Eq. (4) & (5) respectively, to the unsteady heat release have been developed by Fleifil et al.,\(^4\) Ducruix et al.,\(^25\) Cho and Lieuwen,\(^26\) and Preetham et al.\(^27\) In general, these transfer functions involve fairly complex time delayed relationships between the perturbations and the unsteady heat release. Cho and Lieuwen have shown that, in certain cases, these relationships can be cast in terms of the familiar “n-tau” models in the limit of low flame Strouhal number, \(St_f \ll 1\), where the Strouhal number is given by \(St_f = \omega L_f / \dot{u}\).

\[
\frac{\partial Q_{v}(x, \tilde{t})}{\partial \tilde{t}} = n \frac{\bar{Q}}{\bar{u}} \frac{\partial u'(\tilde{t} - \tilde{\tau})}{\partial \tilde{t}} \delta(\tilde{x} - \tilde{x}_f) \tag{4}
\]
\[
\frac{\partial Q_\phi(\bar{x}, \bar{t})}{\partial \bar{t}} = n_\phi \frac{\partial \phi(\bar{t} - \bar{t}_\phi)}{\partial \bar{t}} \delta(\bar{x} - \bar{x}_f) \quad (5)
\]

The unsteady heat release transfer function given by Eq. (4) & (5) are each characterized by two parameters: a time delay and a gain. Delays are used to model the time it takes a disturbance to advect from its point of origin (e.g., fuel injection point, flow separation point) to the flame front and react. Consequently, it contains the sum of time delays from several distinct physical processes.

We assume a proportional, timed delayed, phase shifter using pressure feedback for control implementation. A controller of this type is very common in the experimental literature due to its ease of implementation. Furthermore, the controller’s linearity allows for analytical tractability. The resulting unsteady heat release transfer function that explicitly uses a proportional timed delayed phase shifter is given by Eq. (6).

\[
\frac{\partial Q_\phi(x, \bar{t})}{\partial \bar{t}} = n_\phi \frac{\partial p'(\bar{t} - \bar{t}_f)}{\partial \bar{t}} \delta(\bar{x} - \bar{x}_f) \quad (6)
\]

Once Eqs. (4) & (6) are substituted in Eq. (1), the temporal dynamics of the acoustic modes can be written as a system of ODE’s using a Galerkin expansion. Spatial modes shapes without combustion are used as a set of basis functions to describe the pressure in Eq. (7), where \( \eta \) and \( \psi \) are the temporal and spatial mode shapes respectively.

\[
p' = \bar{p} \sum_{m=1}^{\infty} \hat{\eta}_m(\bar{t}) \psi_m(\bar{x}) \quad (7)
\]

\[
u' = \sum_{m=1}^{\infty} \frac{c^2}{\gamma \omega_m^2} \hat{\eta}_m(\bar{t}) \nabla \psi_m(\bar{x}) \quad (8)
\]

The spatial mode shapes are orthonormal to each other. For the specific calculations shown in this paper, the spatial mode shapes associated with a cylindrical duct with pressure release boundary conditions are used. This means that \( \psi \) belongs to the family of cosine functions to ensure that the velocity perturbations due to acoustics are zero at both ends of the combustor.

For analytical convenience, the series sum in both Eq. (7) & (8) will be truncated to one mode, implying that the influence of other combustor modes upon the controlled mode are neglected. In general, modal coupling in feedback controlled combustors can be significant, as shown by Fleifil et al. Modal truncation appears to be a valid approximation, however, under conditions where truncated modes have significantly higher damping than the fundamental mode. This assumption will require further analysis in future work.

When Eq. (7) & (8) are substituted into Eq. (1), the temporal mode shape can be solved by multiplying the spatial mode shape and integrating over the combustor volume, leading to Eq.(9). While evaluating the integrals the flow boundary conditions are applied using Gauss’ theorem with space and time non-dimensionalized by the combustor length and the period of the fundamental mode respectively. As a final convenience, the temporal mode shape amplitude is normalized by the standard deviation of the additive noise. The non-dimensional variables are donated by the removal of the overhead tilde.

\[
\dot{\eta}(t) + 4\pi \xi \dot{\eta}(t) + 4\pi^2 \eta(t) = \varepsilon_e \eta(t - \tau_e) + \varepsilon_c \dot{\eta}(t - \tau_c) + \xi(t) \quad (9)
\]

With the mean heat release per unit volume approximated as \( \rho u c_T \), the unsteady heat release gains can be written as follows:

\[
\varepsilon_e = -\frac{n_e L_e}{4} \frac{\Delta T}{T} \frac{\partial \psi(x_f)}{\partial x} \psi(x_f) \quad (10)
\]

\[
\varepsilon_c = \frac{\gamma}{2} M \frac{L_e}{L_f} \frac{\Delta T}{T} \psi^2(x_f) \quad (11)
\]

Equation (9) is a dimensionless oscillator equation describing the time evolution of a single mode of the pressure response. The left hand side is a damped acoustic oscillator. On the right hand side are heat release source terms representing the effects of self-excitation and control, respectively. These two time delays can be viewed as competing feedback loops where in the internal loop the self-excitation due to velocity perturbations is destabilizing.
and is competing with the stabilizing outer feedback loop due to control. The final term is additive noise which drives the system.

![Feedback diagram of Eq. (9), showing a controlled, self-excited combustor with additive background noise.](image)

Temporal dynamics of the pressure, $\eta(t)$, are nearly sinusoidal. As such, it is convenient to write the pressure as a sinusoidal signal with a time varying amplitude and phase.  

$$\eta(t) = r(t) \sin \left( 2\pi t + \phi(t) \right)$$  \hspace{1cm} (12)

Since the pressure is rewritten as a function of two random variables in Eq. (12), any arbitrary auxiliary relationship can be specified, see Eq. (13):

$$0 = \dot{r}(t) \sin \left( 2\pi t + \phi(t) \right) + r(t) \dot{\phi}(t) \cos \left( 2\pi t + \phi(t) \right)$$  \hspace{1cm} (13)

$$\dot{\eta}(t) = 2\pi r(t) \cos \left( 2\pi t + \phi(t) \right)$$  \hspace{1cm} (14)

The amplitude and phase can then be written as:

$$r(t) = \sqrt{\eta(t)^2 + \left( \frac{\dot{\eta}(t)}{2\pi} \right)^2}$$  \hspace{1cm} (15)

$$\tan \left( 2\pi t + \phi(t) \right) = \frac{\eta(t)}{\frac{\dot{\eta}(t)}{2\pi}}$$  \hspace{1cm} (16)

Two particular grouping of variables appear in Eq. (15) and Eq. (16) which can be denoted by $X$ and $Y$, respectively, in Eq. (17). This allows the simplification of the expressions for amplitude and phase. For convenience in Eq. (18) the fast and slow varying components for the phase has been lumped together as one term.

$$X(t) = \eta(t) \hspace{1cm} Y(t) = \frac{\dot{\eta}(t)}{2\pi}$$  \hspace{1cm} (17)

$$r(t) = \sqrt{X^2(t) + Y^2(t)} \hspace{1cm} \theta(t) = \tan \left( \frac{X(t)}{Y(t)} \right)$$  \hspace{1cm} (18)

### III. Amplitude-Phase PDF Analysis

The key objective of this paper is to understand the influence of active control upon the combustor dynamics, based upon the solution of Eq. (9). Given that this is a stochastic problem, this is accomplished by developing expressions for the steady-state statistics of the combustor response. For example, the ensemble averaged value of the pressure amplitude, $r(t)$, quantifies the average limit cycle amplitude of the controlled combustor while its variance quantifies the degree of amplitude breathing, discussed in the context of Figure 2. The objective of this section is to derive expressions for these steady state statistics of Eq. (9). This solution procedure can be broken into...
three procedural steps. First, the region of stability of Eq. (9) is solved. Outside of this region, the system is linearly unstable and requires higher order nonlinear effects to describe the limit cycle response.\textsuperscript{23} Within the stable regions, solution properties of linear Gaussian systems are used in conjunction with Fourier analysis to solve for pressure and velocity statistics. A variable transformation is then used to convert the pressure and velocity PDF’s into amplitude and phase PDF’s. It should be noted that this paper works exclusively with the non-unitary Fourier transform convention which is denoted below.

\[
\mathfrak{F} \left[ f(x) \right] = \int_{-\infty}^{\infty} f(x) e^{j\omega x} \, dx \quad \mathfrak{F}^{-1} \left[ f(v) \right] = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(v) e^{-j\omega v} \, dv 
\]  
(19)

A. Stability Boundaries

For the class of SDDE’s driven by additive noise, there exists a stationary solution if the deterministic part of the SDDE is uniformly exponentially stable and the noise is finite variance.\textsuperscript{24} Physically realizable noise terms such as the combustion noise in Eq. (9) must be finite variance. The deterministic part of Eq. (9) can be found by taking its expectation which yields a DDE. In linear time invariant systems, asymptotic stability implies uniform exponential stability, because all characteristic responses are exponentially decaying sinusoids. Asymptotic stability can be determined by the method of harmonic balance, an application of the describing function method assuming a sinusoidal response of unknown frequency, as in Eq. (20). It should be noted that Eq. (20) differs from Eq. (12) because the first order application of the method of harmonic balance is the projection of response on the first Fourier basis function of frequency, \( \omega_{\text{act}} \).

\[
\eta(t) = a \sin \left( \omega_{\text{act}} t \right) 
\]  
(20)

Justifying the use of the method of harmonic balance to solve for regions of stability requires the explanation of two simplifying assumptions: the assumed form of the solution and the neglection of higher order projections in Eq. (20). The characteristic equation of the expectation of Eq. (9) is transcendental and has an infinite number of roots. Each complex pair of roots corresponds to an exponential sinusoidal response because the original equation is linear. Roots on the \( j\omega \)-axis give sinusoidal responses; hence the method of harmonic balance fully captures the behavior of these roots. Additionally, it picks up the Fourier projection of the contribution of all the other roots. Any roots in the open right hand complex plane will add an unstable component to the answer, regardless of the Fourier projection used. Therefore the stability of the system is the same as the stability of the first order harmonic balance solution.

The application of the method of harmonic balance works by substituting the assumed form of the solution for the time delayed terms and using trigonometric identities to rewrite them in terms of \( \eta \) and its derivative. For each root on the \( j\omega \)-axis, there exists an effective ODE describing the contribution of that root to the system response.

\[
\left\langle \dot{\eta}(t) \right\rangle + 2\zeta_{\text{eff}} \omega_{\text{act}} \left\langle \eta(t) \right\rangle + \omega_{\text{act}}^2 \left\langle \eta(t) \right\rangle = 0 
\]  
(21)

where

\[
\omega_{\text{act}}^2 = 4\pi^2 - \varepsilon_x \cos \left( \omega_{\text{act}} \tau_c \right) - \varepsilon_x \omega_{\text{act}} \sin \left( \omega_{\text{act}} \tau_v \right) 
\]  
(22)

\[
\zeta_{\text{eff}} = \frac{2\pi^2 \varepsilon_x}{\omega_{\text{act}}} + \frac{\varepsilon_y \sin \left( \omega_{\text{act}} \tau_c \right)}{2\omega_{\text{act}}^2} - \frac{\varepsilon_y \cos \left( \omega_{\text{act}} \tau_v \right)}{2\omega_{\text{act}}} 
\]  
(23)

Eq. (22) is a transcendental equation for the frequency of oscillations, \( \omega_{\text{act}} \), which has a finite number of roots.\textsuperscript{25} The system response is stable if the damping coefficient is positive for all possible roots. Stability boundaries are then determined from the locus of curves where \( \zeta_{\text{eff}} = 0 \).

B. Solution of Pressure Statistics and its Derivative

This section describes the solution procedure used to derive the solution statistics inside regions of linear stability, under the assumption of Gaussian noise, \( \zeta(t) \). The governing equation can be viewed as an infinite-dimensional linear system that maps a random input \( \zeta(t) \) to a state vector \( [X\ Y]^T \). A linear system responds with Gaussian outputs if it has Gaussian inputs.\textsuperscript{26} This implies that the PDF of the state vector is a bivariate Gaussian that is fully described by five statistics, two from the mean state vector \( \mu \) and three from its covariance matrix \( \Sigma \). A general bivariate Gaussian PDF is given in Eq. (24).

\[
f \left( \tilde{x} = [X\ Y]^T \right) = \frac{1}{2\pi \sqrt{\det \Sigma}} \exp \left\{ -\frac{1}{2} (\tilde{x} - \mu)^T \Sigma^{-1} (\tilde{x} - \mu) \right\} 
\]  
(24)
\[
\Sigma = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy}^2 \\
\sigma_{xy}^2 & \sigma_y^2
\end{bmatrix}
\]  

(25)

With this general form of the solution, it now remains to determine the mean and covariance values. The mean state vector can be shown to equal zero by taking the steady state limit of the expectation of Eq. (9). Individual terms in the covariance matrix can be solved indirectly from the power spectral matrix, \( \Phi \), which is the Fourier transform of the covariance matrix, Eq. (25). The transfer function between \( \Phi_{xx} \) and \( \Phi_{ww} \) is found by taking the Fourier transform of Eq. (9) and multiplying by its complex conjugate. Since the relationship between \( X \) and \( Y \) is known, \( \Phi_{xy} \) and \( \Phi_{yy} \) can be rewritten in terms of \( \Phi_{xx} \).

\[
\Phi_{xx}(v) = \frac{2\pi}{jv} \Phi_{xy}(v) = \frac{4\pi^2}{v^2} \Phi_{yy}(v) = \frac{\Phi_{ww}(v)}{D(v)}
\]

(26)

where

\[
D(v) = (4\pi^2 - v^2)^2 - 2(4\pi^2 - v^2)(v^2) + 8\pi \xi v (\xi - v\cos(\xi v)) + 2 \xi v^3 + \xi v^2
\]

\[
\ldots + 8\pi \xi v^3 (\xi - v\cos(\xi v)) + 2 \xi v^3 + \xi v^2
\]

(27)

The power spectral matrix can be converted to a covariance matrix by taking the inverse Fourier transform; however, only the statistics evaluated at time zero are needed to describe the steady state PDF. Moreover, the power spectrum of any variable must be symmetric. Equation (26) shows that the cross power spectrum between \( X \) and \( Y \) is anti-symmetric. This implies that \( X \) and \( Y \) are always uncorrelated at time zero because the integrand from the inverse Fourier transform of \( \Phi_{xy} \) is anti-symmetric. This leads to the following expressions for \( \sigma_x \) and \( \sigma_y \):

\[
\sigma_x^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{ww}(v) \frac{dv}{D(v)}
\]

(28)

\[
\sigma_y^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} v^2 \Phi_{ww}(v) \frac{dv}{4\pi^2 D(v)}
\]

(29)

The bivariate Gaussian PDF of Eq. (24) then simplifies into:

\[
f(X,Y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp \left[ -\frac{\sigma_x^2 X^2 + \sigma_y^2 Y^2}{2\sigma_x^2 \sigma_y^2} \right]
\]

(30)

### C. Solution for Amplitude and Phase Statistics

The joint PDF of the pressure and velocity in Eq. (30) can be used to derive an amplitude-phase PDF via the Jacobian of the transform.\(^34\) This accounts for the change of area in differential elements of the PDF when converting \( X \) and \( Y \) into \( r \) and \( \theta \). The amplitude-phase PDF is given by Eq. (32).

\[
J(r,\theta) = \left| \begin{array}{c}
\frac{\partial X}{\partial r} & \frac{\partial X}{\partial \theta} \\
\frac{\partial Y}{\partial r} & \frac{\partial Y}{\partial \theta}
\end{array} \right| = \frac{1}{r}
\]

(31)

\[
f(r,\theta) = \frac{f(X(r,\theta),Y(r,\theta))}{J(r,\theta)} = \frac{r}{2\pi \sigma_x \sigma_y} \exp \left[ -\frac{(\sigma_x^2 \cos^2 \theta + \sigma_y^2 \sin^2 \theta) r^2}{2\sigma_x^2 \sigma_y^2} \right]
\]

(32)

In the limiting case of no time delays \( \sigma_x = \sigma_y = \sigma_{xy} \) and \( r(t) \) and \( \theta(t) \) are independent of each other. Moreover, the phase PDF is uniformly distributed and the amplitude PDF is a Rayleigh distribution given by Eq. (33). This amplitude PDF is fully described by the Rayleigh parameter in Eq. (34). Note that \( \sigma_{xy} \) has the physical interpretation as the amplitude at which the PDF has its maximum value (i.e., the most likely amplitude).
\[
f(r, \theta) = \frac{r}{2\pi \sigma_{xy}^2} \exp \left[ -\frac{r^2}{2\sigma_{xy}^2} \right]
\]

\[
\sigma_{xy} = \sqrt{\frac{1}{32\pi^3 \zeta}}
\]

Equation (33) serves as a useful comparison when investigating the effects of time delays on the amplitude and phase statistics. The marginal amplitude and phase PDF’s of the time delayed system can be determined from Eq. (32) to yield:

\[
f(\theta) = \frac{\sigma_x \sigma_y}{2\pi} \left( \sigma_x^2 \sin^2(\theta) + \sigma_y^2 \cos^2(\theta) \right)^{-1}
\]

\[
f(r) = \frac{2\pi}{2\pi \sigma_x \sigma_y} \exp \left[ -\frac{\left( \sigma_y^2 \cos^2(\theta) + \sigma_x^2 \sin^2(\theta) \right) r^2}{2\sigma_x^2 \sigma_y^2} \right] d\theta
\]

A key effect of time delays is to cause \( \sigma_x \) and \( \sigma_y \) to differ in value. In the marginal phase PDF, this leads to regions of preferential phase as seen in Figure 4, as opposed to the non time-delayed systems where all phases are equally probable. The amplitude of these undulations in the phase PDF is controlled by the ratio of \( \sigma_x/\sigma_y \). If this ratio is less than one, then the most likely phase is \( \pi/2 \) and \( 3\pi/2 \), while it is \( \pi \) and \( 2\pi \) when this ratio is greater than one. From a control point of view, these optimal phases and multiples of \( 2\pi \) correspond to the set of optimal controller time delays. One set is valid when the controller gain is positive while the other is valid for negative gain.

\[\text{Figure 4. Effects of the standard deviation ratio, } \sigma_x/\sigma_y, \text{ on the ratio of the standard deviation to the mean of the pressure amplitude.}\]

\[\text{Figure 5. Marginal PDF of pressure phase, } \phi(t), \text{ plotted against the standard deviation ratio, } \sigma_x/\sigma_y.\]

The marginal amplitude PDF in Eq. (36) cannot be analytically evaluated; however, expressions can be derived for its statistics. Of particular interest are the first two moments of amplitude shown below, where \( E \) represents the incomplete elliptical integral of the second kind.

\[
\langle r \rangle = \frac{\sigma_y}{\sqrt{8\pi}} \int_0^{2\pi} \sqrt{1 - \left( 1 - \frac{\sigma_x^2}{\sigma_y^2} \right) \sin^2(\theta)} d\theta = \frac{\sigma_y}{\sqrt{8\pi}} E \left( 2\pi, 1 - \frac{\sigma_x^2}{\sigma_y^2} \right)
\]

\[
\langle r^2 \rangle = \sigma_x^2 + \sigma_y^2
\]

Amplitude breathing can be characterized by the ratio of the standard deviation to the mean, \( c_r \). The dependence of \( c_r \) upon \( \sigma_x/\sigma_y \) is plotted in Figure 5. Calculated ranges of realizable \( \sigma_x/\sigma_y \) based upon Eq. (37) & (38) were found to never be less than unity and varied by no more than a few percent. The sole exception is on the very edge of instability where \( c_r \) reaches values close to \( \sqrt{2} \). Otherwise, this implies a roughly constant predicted \( c_r \) value of \( \sqrt{(4\pi)/\pi} \). In contrast, experimental observations appear to indicate significant variations in \( c_r \), generally with this

American Institute of Aeronautics and Astronautics
parameter increasing as the instability amplitude is suppressed.\textsuperscript{2} We suspect that this may be due to observer
dynamics and indicates that incorporation of these effects is an important next step to the work described here.

### IV. Stability and Performance Maps

This section uses the previously described solutions to quantitatively plot controlled system performance as a
function of heat release gains, time delays, and damping coefficient. The primary focus is placed upon understanding trends in best possible controlled combustor characteristics. Nominal system characteristics without
control are considered first before investigating the effects of control. Explicit calculation of $\sigma_i$ and $\sigma_i$ requires
specification of the noise power spectrum. We assume white noise for analytical simplicity. Other colored noise
spectrum will give essentially the same results as long as they are spectrally flat in the vicinity of the dimensionless
frequency of oscillations, $2\pi$.

#### A. Uncontrolled Combustor

Figure 6 plots the dependence of the inverse mean limit cycle amplitude, $1/\langle r \rangle$, upon the heat release gain and
time delay. In time delay intervals of about 0.5, the system alternates between regions of stability and instability,
assuming the gain is large enough to overcome the inherent system damping. This is a classic result (e.g., see
McManus et al.\textsuperscript{35}); i.e., that instabilities occur in certain time delay bands associated with the correct phasing
between pressure and heat release to satisfy Rayleigh’s criterion. However, stability bands shrink with increasing
gain or very large time delays. The reason for the loss of stability is that large time delays excite multiple
frequencies and, at some point, one of them is always unstable.

![Figure 6. Limit cycle amplitude of an uncontrolled self-excited combustor with $\zeta = 0.04$](image)

![Figure 7. Limit cycle amplitude of an uncontrolled self-excited combustor with $\zeta = 0.08$](image)

Inspection of Figure 6 also shows that there is a band of system gain values in which the system is always stable
for all time delays and another value at which the system is always unstable. These behaviors are called delay-
indepedent stability and delay-independent instability respectively. These gain bands depend on the damping
coefficient of the system. Niculescu\textsuperscript{36} shows that when $\zeta \leq 1/\sqrt{2}$ the system is delay-independent stability for $|c_v| \leq 8\pi^2\zeta^2(1-\zeta^2)$. Delay-independent instability is exhibited when $c_v \geq 4\pi^2$.

Increases in value of the damping parameter, $\zeta$, cause a decrease in mean limit cycle amplitude, while pushing the stability boundaries
outwards in the direction normal to the mean amplitude iso-contours. This is illustrated when comparing Figure 6 with Figure 7. However,
variations in the damping coefficient do not fundamentally change the system behavior when $\zeta \ll 1$ and, hence, they are not further explored
in this paper. Rather, the rest of the paper focuses on gain and time delay influences on controlled combustor performance. In the next
subsections, the influence of closed loop control is considered with
respect to the results shown in Figure 6. In order to reduce the number of degrees of freedom from four ($c_v$, $\tau_v$, $c_v$, $\tau_v$) to two ($c_v$, $\tau_v$), focus is
placed upon four test points shown in Table 1: two nominally stable and
two nominally unstable.

<table>
<thead>
<tr>
<th>Label</th>
<th>$\zeta$</th>
<th>$\varepsilon_v$</th>
<th>$\tau_v$</th>
<th>Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.04</td>
<td>-6</td>
<td>0.75</td>
<td>Yes</td>
</tr>
<tr>
<td>U1</td>
<td>0.04</td>
<td>-6</td>
<td>1.25</td>
<td>No</td>
</tr>
<tr>
<td>S2</td>
<td>0.04</td>
<td>-6</td>
<td>1.75</td>
<td>Yes</td>
</tr>
<tr>
<td>U2</td>
<td>0.04</td>
<td>-6</td>
<td>2.25</td>
<td>No</td>
</tr>
</tbody>
</table>
B. Nominally Stable Combustor with Control

Nominally stable test points, S1 and S2, are investigated first. Results are plotted in Figure 9 and Figure 10, respectively. The key difference between these two test points is the internal time delay, $\tau_c$. The objective of this study is to assess the extent to which closed loop control can reduce the amplitude of stable, noise driven oscillations of a lightly damped system. Note that the axes on these graphs correspond to the gain and time delay of the controller. For small controller delay, approximately $\tau_c < 2$, it can be seen that it is possible to lower the mean limit cycle amplitude. As the internal delay is increased from Figure 9 to Figure 10, it becomes more difficult to obtain amplitude reductions with control. The feedback gains must increase and the range of allowed controller delays decrease.

Despite the decrease in performance from increasing the internal delay, it becomes easier to find regions of improved performance for large controller delays; however, these improvements can be quite marginal. The associated minimum limit cycle amplitude is plotted in Figure 8, which shows that the increase in internal delays creates undulations in the minimum limit cycle amplitude which allows a control system to extract performance gains for a nominally stable system. For large enough control delay, the best controller is one that does nothing, indicated by comparison with the dashed “no control” lines. This line is approached at smaller control delays as the internal delay decreases. Any attempt at control in this case worsens system performance.

C. Nominally Unstable Combustor with Control

Consider next the test points U1 and U2, corresponding to conditions where the combustor is nominally unstable. There are two figures of merit for these cases: the regions in controller gain-time delay space where the system can be stabilized, and the associated limit cycle amplitudes. Results are shown in Figure 11 and Figure 12 respectively. Each figure has isolated regions where the system is stable that are alternatively spaced between positive and negative control gains with intervals of approximately 0.5 in control delay. Throughout this section, these regions will be referred to as stability islands. On a nominally unstable self-excited system, it can be seen that the size of the stability islands decrease as the control time delay increases; however, this does not globally hold as the internal time delay increases. Moreover, note the general trend that the minimum limit cycle amplitude of the controlled combustor within a given stability island also increases with increased control time delay.
Figure 13 shows how the minimum possible limit cycle amplitude changes with the stability island centroid control time delay over a range of internal delays. Additionally, the size of each marker is representative of the size of the stability island. Note that the general notion of increasing time delays increasing system tendencies towards instability must be nuanced in the case with large internal delays. Rather, in these cases, either small time delays, or time delays that are close to internal time delays appear to produce the best controlled combustor behavior. The reader is reminded that the time delays show up in different derivatives of \( \eta \) if they were in the same order of derivative, the control would identically cancel out the internal feedback loop. As the internal delay increases, the stability island’s performance and size begin to oscillate. This suggests that adding artificial delays to the controller to make them commensurate with the value of the internal delay may produce better controlled performance. However, this observation requires further analysis where the additional effects of observer dynamics are incorporated. What is clear from this graph is that the same active control system (parameterized here by its gain and delay) can produce dramatically different influences on limit cycle amplitudes of the controlled combustor, depending upon the internal time delays. This observation appears to explain many experimental observations of similar behavior.

V. Concluding Remarks

This paper presents exact solutions and analysis of the statistics of a self excited combustor with closed loop feedback control. In particular, we discuss the influence of controller gain and time delay upon regions over which the system can be stabilized, as well as the limit cycle amplitude of the controlled combustor. In addition, we discuss the degree of amplitude breathing of the controlled combustor. The model predictions are consistent with the
experimental observation that the same control system can have dramatically different effects upon controlled combustor amplitude on different combustors or at different operating conditions. Within the framework of this model, this was demonstrated by showing that the effect of the same controller is quite different for different unstable combustors with different internal time delays. The model also predicts, however, that the degree of amplitude breathing is relatively modest and almost invariant with control authority. This latter prediction does not appear consistent with qualitative experimental observations and provides clues where additional physics are needed.

We believe that this discrepancy is due to the additional degrees of freedom associated with observer dynamics. As such, incorporating these observer dynamics, as well as modal coupling processes, is a key focus of our ongoing work. In addition, steady state statistics are able to show what the control system does but not how it does it. A particularly interesting loose end is how a control system interacts with combustion physics to create regions of preferential phase. Analyses of transient system statistics will be useful toward this end.

### Appendix

#### Table 2: Experimental AIC Results by Secondary Fuel Injection

<table>
<thead>
<tr>
<th>Reference</th>
<th>Temp</th>
<th>Press</th>
<th>Fuel</th>
<th>Controller</th>
<th>Freq.</th>
<th>Reduc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Park et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Ethylene/Ethanol</td>
<td>Posicast</td>
<td>38 Hz</td>
<td>28 dB</td>
</tr>
<tr>
<td>Murugappan et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Ethanol</td>
<td>LQG-LTR</td>
<td>205 Hz</td>
<td>14 dB</td>
</tr>
<tr>
<td>Anson et al.</td>
<td>600 K</td>
<td>6.5 barr</td>
<td>Natural Gas</td>
<td>Adaptive Phase</td>
<td>50 Hz</td>
<td>5 dB</td>
</tr>
<tr>
<td>Yu and Wilson</td>
<td>25°C</td>
<td>1 atm</td>
<td>JP-10</td>
<td>Phase Delay</td>
<td>125 Hz</td>
<td>15 dB</td>
</tr>
<tr>
<td>McManus et al.</td>
<td>400 K</td>
<td>2 MPa</td>
<td>Heptane</td>
<td>Open Loop</td>
<td>140 Hz</td>
<td>6 dB</td>
</tr>
<tr>
<td>McManus et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Heptane</td>
<td>Time Delay</td>
<td>75 Hz</td>
<td>6 dB</td>
</tr>
<tr>
<td>Le et al.</td>
<td>770°F F</td>
<td>200 psi</td>
<td>JP-8</td>
<td>MSEEK</td>
<td>535 Hz</td>
<td>3 dB</td>
</tr>
<tr>
<td>DeLaat and Chang</td>
<td>775°F F</td>
<td>175 psi</td>
<td>JP-8</td>
<td>Adaptive Phase</td>
<td>535 Hz</td>
<td>3 dB</td>
</tr>
<tr>
<td>Kopasakis et al.</td>
<td>770°F F</td>
<td>200 psi</td>
<td>JP-8</td>
<td>Adaptive Phase</td>
<td>535 Hz</td>
<td>4.5 dB</td>
</tr>
<tr>
<td>Le et al.</td>
<td>770°F F</td>
<td>200 psi</td>
<td>JP-8</td>
<td>STR MSEEK</td>
<td>315 Hz</td>
<td>20 dB</td>
</tr>
<tr>
<td>Riley et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Ethylene</td>
<td>STR</td>
<td>207 Hz</td>
<td>26 dB</td>
</tr>
<tr>
<td>Evesque et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Ethylene</td>
<td>STR</td>
<td>207 Hz</td>
<td>26 dB</td>
</tr>
<tr>
<td>Jones et al.</td>
<td>415°C</td>
<td>14.7 psi</td>
<td>Methane</td>
<td>Phase Delay</td>
<td>350 Hz</td>
<td>22 dB</td>
</tr>
<tr>
<td>Shcherbik et al.</td>
<td>17°C</td>
<td>718 psi</td>
<td>Natural Gas</td>
<td>Open Loop</td>
<td>375 Hz</td>
<td>20 dB</td>
</tr>
<tr>
<td>Sattinger et al.</td>
<td>332 K</td>
<td>125 psi</td>
<td>Natural Gas</td>
<td>Observer</td>
<td>230 Hz</td>
<td>12 dB</td>
</tr>
<tr>
<td>Butts et al.</td>
<td>300°C</td>
<td>450 psi</td>
<td>Natural Gas</td>
<td>Adaptive Phase</td>
<td>420 Hz</td>
<td>6 dB</td>
</tr>
<tr>
<td>Lubarsky et al.</td>
<td>25°C</td>
<td>597 psi</td>
<td>Natural Gas</td>
<td>Open Loop</td>
<td>370 Hz</td>
<td>20 dB</td>
</tr>
<tr>
<td>Johnson et al.</td>
<td>200°C</td>
<td>14.7 psi</td>
<td>Heptane</td>
<td>Adaptive Phase</td>
<td>400 Hz</td>
<td>4.5 dB</td>
</tr>
<tr>
<td>Cohen et al.</td>
<td>730 K</td>
<td>1.56 MPa</td>
<td>Diesel</td>
<td>STR</td>
<td>200 Hz</td>
<td>15 dB</td>
</tr>
<tr>
<td>Seume et al.</td>
<td>1500 C</td>
<td>17 bar</td>
<td>Natural Gas</td>
<td>Phase Delay</td>
<td>433 Hz</td>
<td>17 dB</td>
</tr>
<tr>
<td>Yu et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Heptane</td>
<td>Phase Delay</td>
<td>35 Hz</td>
<td>15 dB</td>
</tr>
<tr>
<td>Murugappan et al.</td>
<td>25°C</td>
<td>5 atm</td>
<td>Ethanol</td>
<td>Extremum-Seeking</td>
<td>220 Hz</td>
<td>4.4 dB</td>
</tr>
<tr>
<td>Coker et al.</td>
<td>430 K</td>
<td>4.4 atm</td>
<td>Jet A</td>
<td>Fuzzy Logic</td>
<td>93 Hz</td>
<td>1.3 dB</td>
</tr>
<tr>
<td>McManus and Magill</td>
<td>25°C</td>
<td>1 atm</td>
<td>Heptane</td>
<td>Time Delay</td>
<td>70 Hz</td>
<td>6 dB</td>
</tr>
<tr>
<td>Hibshman et al.</td>
<td>730 K</td>
<td>1.1 MPa</td>
<td>Diesel</td>
<td>Phase Delay</td>
<td>200 Hz</td>
<td>6.5 dB</td>
</tr>
<tr>
<td>Sivasegaram et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Kerosene</td>
<td>Open Loop</td>
<td>120 Hz</td>
<td>12 dB</td>
</tr>
<tr>
<td>Langhorne et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Ethylene</td>
<td>Time Delay</td>
<td>74 Hz</td>
<td>14 dB</td>
</tr>
<tr>
<td>Acharya et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Ethanol</td>
<td>Phase Delay</td>
<td>200 Hz</td>
<td>14 dB</td>
</tr>
<tr>
<td>Kopasakis et al.</td>
<td>770°F F</td>
<td>200 psi</td>
<td>JP-8</td>
<td>Adaptive Phase</td>
<td>315 Hz</td>
<td>31 dB</td>
</tr>
<tr>
<td>Riley et al.</td>
<td>25°C</td>
<td>1 atm</td>
<td>Ethylene</td>
<td>STR</td>
<td>244 Hz</td>
<td>30 dB</td>
</tr>
<tr>
<td>Guyot et al.</td>
<td>400 K</td>
<td>1 atm</td>
<td>Natural Gas</td>
<td>Open Loop</td>
<td>95 Hz</td>
<td>10 dB</td>
</tr>
<tr>
<td>Lacarelle et al.</td>
<td>298 K</td>
<td>1 atm</td>
<td>Natural Gas</td>
<td>Extremum-Seeking</td>
<td>95 Hz</td>
<td>20 dB</td>
</tr>
<tr>
<td>Kim et al.</td>
<td>298 K</td>
<td>1 atm</td>
<td>Natural Gas</td>
<td>Phase Delay</td>
<td>360 Hz</td>
<td>22 dB</td>
</tr>
<tr>
<td>Cohen et al.</td>
<td>710 K</td>
<td>1.5 MPa</td>
<td>Natural Gas</td>
<td>Thresh Holding</td>
<td>200 Hz</td>
<td>16 dB</td>
</tr>
</tbody>
</table>

13

American Institute of Aeronautics and Astronautics
Acknowledgments

This work has been supported by NASA under cooperative agreement NNX07AC92A under Dr. John DeLaat and Clarence Chang as technical monitors.

References


American Institute of Aeronautics and Astronautics