The K-H Instability of Reacting, Acoustically Excited Bluff-Body Shear Layers

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This paper describes an experimental study of the effect of acoustic excitation on bluff-body stabilized flames. The Kelvin-Helmholtz (KH) instability of the shear layer is excited due to the incident acoustics. In turn, the KH instability imposes a convecting, harmonic excitation on the flame, which leads to spatially periodic flame wrinkling and heat-release oscillations. Understanding the factors influencing these heat release oscillations therefore requires an understanding of the generation, convection, and dissipation of these vortical disturbances. The evolution of these vortical disturbances is strongly influenced by the presence of combustion due to enhanced diffusivity in the hot products, volume dilatation, and baroclinic torque.

PIV measurements are reported of the decay of these vortices over a range of conditions, which suggest that the high product diffusivity controls the reduction in vorticity amplitude downstream. Of particular significance is the relative location of the flame and vortex sheet. If the vortex sheet is inside the hot products, it dissipates much more rapidly than if it lies in the reactants. In addition, experiments were performed with two bluff bodies, one with a triangular cross section and another with a circular cross section. The triangle has a well-defined separation point, leading to phase locked and transversely symmetric vorticity and flame wrinkling. In contrast, while instantaneous images from the circular bluff body look similar to those of the triangle, overlays from cycle to cycle reveal a substantial amount of phase jitter in the vortex sheet, and therefore flame position. Vorticity fluctuations of comparable magnitudes are generated instantaneously for both bluff body shapes, but the spatial jitter leads to reduced ensemble averaged amplitudes for the circular bluff body. This phase jitter is reduced by increasing the amplitude of acoustic excitation.

I. Introduction

The objective of this research is to explore the dynamical processes controlling the response of bluff-body stabilized flames to harmonic oscillations. It is motivated by the problem of combustion instabilities, which arise due to complex feedback process between pressure, velocity and heat-release oscillations. Both the flame and the flow are influenced by acoustic excitation; e.g., shear layer rollup and/or vortex shedding frequency events lock into the forcing. As such, understanding the problem of interest requires understanding these interactions and their relative significance.

There are three key issues of interest here. The first is the response of the flame to harmonic excitation. Substantial contributions have been made towards understanding the interaction between harmonic waves and premixed flames, and a host of factors have been identified as significant, such as stabilization dynamics and the spatial character of the disturbance field. The second point of interest is the response of the flow-field to imposed acoustic oscillations. Studies have shown that harmonic flow oscillations influence the characteristics of absolute and convective instabilities already present in bluff body flowfields, which have their respective frequency/amplitude response characteristics. The third issue is the influence of the flame on the flowfield. For example, baroclinic vorticity and volume dilatation effects alter the downstream evolution of the flow oscillations at the flame. The following subsections treat each of these issues in more detail.

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A. Response of the flame to harmonic excitation

A number of prior studies have characterized the interaction of flames with harmonic waves arising due to both acoustic waves and also convecting, vortical disturbances. The dynamics of the flame is controlled by flame kinematics, i.e., the propagation of the flame normal to itself at the local burning velocity, and the local flow field that the flame is locally propagating into. This is mathematically described by the so-called G equation:

\[ \frac{\partial G}{\partial t} + \mathbf{u} \cdot \nabla G = S_L |\nabla G| \]  

(1)

In this equation the flame position is described by the parametric equation \( G(\bar{x},t) = 0 \). Also, \( \mathbf{u} = \bar{u}(\bar{x},t) \) and \( S_L \) denote the flow field just upstream of the flame and laminar burning velocity, respectively. In the unsteady case, the flame is being continually wrinkled by the unsteady flow field, \( \bar{u} \). The action of flame propagation normal to itself, the term on the right side of Eq. (1), is to attempt to smooth these wrinkles out. As such, a wrinkle created at one point of the flame due to a velocity perturbation propagates downstream and diminishes in size due to flame propagation. Indeed, the dynamical interaction between the driving (acoustic oscillations) and the damping (restoration property of the flame) can lead to a range of effects depending upon the relative values of the flow oscillations and flame speed. This manifests itself through both local influences upon the flame topology (e.g., cusping, amplitude of corrugation, pocket formation), and global influences upon the overall unsteady heat release response of the flame.

B. Response of the bluff body flow-field to oscillations

The bluff-body flow-field consists of a boundary layer, a separated shear layer and a wake. The velocity field of the separated shear layer and the wake has an inflexion point, rendering them susceptible to hydrodynamic instabilities. The wake mode, referred to here as the Bénard/von Kármán instability (BvK), leads to alternate shedding of vorticity from opposite sides of the bluff body and a sinuous wake structure. The frequency of this instability scales \( f_{BVK} = S_t U_0 / D \).

The shear layer instability, or Kelvin-Helmholtz (KH) instability, is a convective instability associated with the amplification of disturbances, leading to vortex rollup and pairing. The frequency of the most amplified KH instability mode is different than that of the BvK, because the appropriate length scale is the shear layer thickness rather than the bluff body size. As such, for high Reynolds number flows the KH instability frequency is much larger than the BvK instability; e.g., relations from Prasad and Williamson lead to \( f_{KH} = 0.0235 f_{BVK} Re^{0.67} \). Under the influence of harmonic excitation, the separated shear layer rolls up into vortices with a frequency commensurate with the frequency of excitation. In addition, due to nonlinear interactions, velocity fluctuations are observed at sum and difference frequencies of the forcing frequency and its harmonics. As discussed in the next section, due to the apparent suppression of the BvK instability in flames with burned to unburned gas temperature ratios greater than about two, the shear layer instability is of particular significance in controlling the dynamics of acoustically excited, bluff body flames. This observation has been corroborated by a number of flow visualizations and calculations.

C. Effect of heat-release on the flow-field

Heat release substantially influences the flow disturbances which are, in turn, disturbing the flame. As an example of such an influence, consider the stabilization of the wake mode instability by volume dilatation as was suggested by Erikson et al. In the absence of any combustion (equivalent to dilatation ratio of 1) the wake mode instability is clearly seen. However as the dilatation ratio increases, the strength of the shed vortices decreases, until at a dilatation ratio of about two, the wake mode instability is apparently absent.

The physics of heat release influences on the flow field can be understood from the vorticity transport equation:

\[ \frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla)\vec{V} - \vec{\omega}(\nabla \cdot \vec{V}) - \frac{\nabla p \times \nabla \rho}{\rho^2} + \nu \nabla^2 \vec{V} \]

Vortex Stretching Gas Expansion Baroclinic Production Viscous Diffusion

(I) (II) (III) (IV)

This equation explains the evolution of the vorticity of a fluid element as it moves in space. Terms I and IV are the vortex stretching term and viscous diffusion term respectively. These terms exert an influence regardless of whether...
combustion occurs in the flow or not. The presence of combustion introduces three notable influences in the flow-field as will be described now.

First, the kinematic gas viscosity (term IV above) sharply rises through the flame, due to its large temperature sensitivity. This will tend to enhance the rate of diffusion and damping of vorticity, an effect emphasized by Coats. Second, due to the inclination of the flame with respect to the flow (and, therefore, the pressure gradient), vorticity is generated by the baroclinic mechanism (term III above), due to the misaligned pressure and density gradients. This vorticity is of the opposite sign as bluff-body-wall generated vorticity, but the same sign as channel-wall generated vorticity, if the body is confined in a channel. As such, there is a competition between the former two vorticity sources, which can result in complete cancellation, and then sign reversal, of flow vorticity in certain regions of the flow. Third, there is generally significant gas expansion behind the flame (term II above). The flow dilatation acts as a vorticity sink, as can be seen by the negative sign of this term, i.e. $\omega(\nabla \cdot V)$ in the vorticity transport equation above.

Fundamental studies of vortex-flame interactions has shown that the nature of the flow changes substantially with the amplitude of vorticity perturbation. For low vortex strengths, the flame is wrinkled with an amplitude proportional to the ratio of $u_0/S_c$, where $u_0$ in this case denotes the velocity amplitude associated with the vortex. The amplitude of the vortex decreases through the flame due to volume dilatation and the large diffusivity of the products. Furthermore, if the flow and flame are nominally normal to each other, the baroclinic term is zero. As the amplitude of the vortex increases, the flame becomes highly wrinkled to the point that vorticity can also be produced/destroyed by baroclinic processes. i.e., very strong vortices distort the flame to such an extent that they change the sign of the baroclinically generated vorticity along the corrugated flame sheet. This point was made by Louch and Bray: “with increasing vortex rotation rates, the flame wraps around the vortex and results in ‘scrambling’ the alignment of the density and pressure gradients, causing different signs of baroclinic torque along the flame”. However, one key difference between the present investigation and the above cited studies is the fact that the flame is nominally at an angle to the flow, so that baroclinic vorticity is present even in the nominal, unforced case. Nonetheless, this discussion illustrates the complications that can arise between shear generated vorticity that has its own dynamics (e.g., rollup, pairing, growth, etc.), viscous diffusion, volume dilatation, and baroclinic processes.

Figure 1. Schematic of interactions associated with acoustic wave excitation of bluff body stabilized flames.

Because of the strong interactions between the flame and flowfield, studying the problem of acoustic wave interactions with bluff body flames does not really allow breaking this problem into the more simple sub-problems, such as “direct” acoustic wave interactions with the flame (because the flame response is most strongly dominated by the vorticity fluctuations excited by the acoustics), or acoustic wave interactions with a non-reacting bluff body flow field (because volume dilatation associated with the flame fundamentally changes the nature of the interaction).

With this background, we seek to obtain a better understanding of the interaction between harmonic waves and the bluff-body flow-field with combustion. As was just discussed, the influence of heat release on the flow introduces various pathways for the interactions between the acoustic waves, flow-field and the flame that are illustrated in Figure 1. The choice of the phrase ‘harmonic waves’ over ‘acoustic waves’ was intentional despite the fact that the excitation source used in the experiments (as will be described next) is acoustic. This allows us to account for the possibility of acoustic waves leading to periodic vortical structures that in turn perturb the flame.

II. Details of Experimental Apparatus

Experiments were carried out in an atmospheric pressure burner with a square cross-section (3.75" x 3.75") that is 3’ long (Figure 2, left). Natural gas and air are introduced in a mixing chamber located at the base of the burner. The
air, fuel, and seeding flow rates are measured with rotameters (purchased from Dwyer Inc.) having ranges spanning 10-100 SCFM, 25-400 SCFH, and 60-600 SCFH, respectively, with accuracies of 2%. Aluminum oxide (Al₂O₃) is used as seed, with particle size ranging from 0.9 – 2.2 µm. A cyclone seeder (not pictured) is used to introduce the seeding particles into the air flow and is mixed with the main flow in the mixing chamber at the bottom of the burner. The seeded fuel-air mixture exits the mixing chamber into a six inch long tube of the same cross-section as the burner, which also contains two 100 Watt Walsch PA acoustic loudspeakers. The mixture then passes through a honeycomb grid flow straightening section, beyond which it flows all the way up to the exit of the channel. The turbulent intensities in the approach flow were about 6%. The acoustic excitation was carried out using loudspeakers that were sinusoidally forced with an Agilent 33120A-15 MHz function generator connected in series with a RadioShack MPA-101, 100 Watt amplifier. The bluff body is mounted at the immediate exit of the channel. Two types of bluff bodies are used, one with a circular cross section and another with a triangular cross section. For experiments in which combustion was absent, a splitter plate was used to suppress the wake mode instability. Diagrams of the bluff bodies and splitter plates are also shown in Figure 2 (right).

Figure 2. (LEFT) Schematic of the experimental setup. (RIGHT) Shapes and dimensions of the (1) triangular and (2) cylindrical bluff body, (3) triangular bluff body with splitter plate. All dimensions are in millimeters.

The velocity field was characterized with particle image velocimetry (PIV). The light source used was a dual head Nd:YAG laser of wavelength 532 nm with a peak power output of 120mJ/pulse. The interval between the beam pulses was set to 30 µs. The light sheet was generated using two cylindrical lenses of 150 mm and 1000 mm focal length. The latter was used to reduce the thickness of the beam and the former was used to diverge the 5 mm laser beam to a height of 40 mm. A 1600x1200 pixels CCD camera, with an F-mount Nikon 55mm micro-lens with an aperture of f/5.6 was used for imaging. The distance between the imaging plane and the camera was set at 12 inches. Phase synchronization of the excitation signal with the PIV system was managed by a LaVision timing generator. At each phase, 128 images were recorded and the resulting data was ensemble averaged to provide data (velocity and vorticity) repeatable to within 2% of its reported value.

Experiments were performed at an approach velocity $U_\infty = 2.7$ m/s and a frequency of excitation $f = 300$ Hz. The amplitude of acoustic excitation, measured at the exit plane of the burner with the bluff body removed, was varied between $0 < u' / U_\infty < 0.3$.

III. Results and Discussion

Previous work has shown that acoustic excitation influences the KH instability of the separated shear layer of the bluff body that, in turn, distorts the flame. Figure 3 illustrates a typical image showing this interaction. The strong vorticity in the bluff body near-field (within 2.5 diameters) associated with the vortex sheet rollup is clearly evident in the convolution of the flame, which is strongly wrapped around the vortex. Due to the flame propagation
As can be seen from the above discussion, the initial amplitude and downstream evolution of the vorticity created in the bluff body shear layer plays a key role in the downstream evolution of the flame wrinkling, and therefore in its global heat release response to perturbations. For example, the rate of decay of the vortical disturbances downstream will play an important role in the decay region of the flame response. Furthermore, the trajectory of these disturbances relative to the flame sheet location also influences the amplitude of flame sheet response for a given vortical disturbance amplitude. The measurements reported in this section were obtained to further quantify the evolution of the vorticity field and its interactions with the flame.

Figure 3. (LEFT) Illustration of the flame under the influence of harmonic excitation\textsuperscript{39}. (RIGHT) Amplitude of fluctuation of flame sheet, $L'$, at each axial location, $x$, in the direction transverse to the flow. Data taken at a lower amplitude velocity fluctuation than shown on left, so that flame position is single-valued function of downstream coordinate, $x/\lambda_c$, where $\lambda_c = U_\infty / f$ is the convective wavelength and $f$ is the frequency of excitation.

The flame dynamics under these circumstances were quantified in a previous work\textsuperscript{32,29}, where it was shown that the amplitude of flame front position fluctuation, $L'$, at the forcing frequency first increased with downstream distance from the bluff body, reached a maximum, and then decreased to essentially zero (Figure 3, right). It was suggested that this behavior was controlled by flame stabilization in the initial, growth region. That is, because the flame is anchored, its amplitude of fluctuation is essentially zero at this point, and $L'$ initially grows with downstream distance. As long as vortical fluctuations are present in the flow, the flame will exhibit wrinkling due to these perturbations. However, the vortical disturbances decay rather quickly downstream, meaning that the source of flame excitation is gone. Also, flame propagation normal to itself is continuously present as a mechanism for smoothing out the wrinkles. In the bluff body nearfield, the amplitude of velocity fluctuations is sufficiently larger than the flame speed, so that this kinematic restoration mechanism is not significant. Farther downstream, however, the velocity fluctuations are of much smaller amplitude, so that the flame quickly smoothes out the wrinkles created in the bluff body nearfield. This decay in amplitude of flame fluctuations can be seen in the $x/\lambda_c > 2$ region in Figure 3.

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A. Analysis of the Vorticity Field

![Figure 4. Illustration of the interaction between the vorticity field and the flame. (TOP LEFT) Phase averaged vorticity field and overlay of instantaneous flame location for 128 images taken at 0° phase acoustic excitation. Instantaneous vorticity field and flame edge at phases 0° (TOP RIGHT), 45° (BOTTOM LEFT), and 90° (BOTTOM RIGHT). Dimensions in the figure are in meters. The flow direction is from left to right.](image)

In order to understand the evolution of the velocity field and flame, Figure 4 plots phase locked images of the flame sheet and vorticity field at three phases. The flow direction is from left to right. The contour (color) plot maps the vorticity field. The black lines illustrate the position of the flame sheet. The first image overlays 128 flame images obtained at this same phase, as well as the ensemble averaged vorticity field. The remaining three phases overlay the instantaneous vorticity and flame position. Examination of the plots illustrates the formation of regions of intense vorticity at the bluff body separation point that subsequently convect downstream. These are presumably associated with the rollup of the harmonically pulsing vortex sheet originating at the separation point. The rotating flow strongly distorts the flame, causing it to be wrapped around the center of vorticity. The fluctuations in vorticity cause corresponding fluctuations in surface area, and therefore, heat release rate of the flame. The decay in vorticity field farther downstream is also clearly evident in the figure – note that the vortex formed in the immediate cycle of forcing is clearly present, the one formed in the prior cycle of forcing (the one associated with the second wrinkle on the flame) is nearly gone but unrecognizable, and the one forced two preceding cycles earlier is not recognizable. This means that the vortex decays within two convective wavelengths for this result; in other results shown later it persists up to four convective wavelengths. Also, note the nearly symmetric response of the shear layer, as evidenced by the nearly identical vorticity and flame sheet on both the top and bottom sides of the bluff body shear layer.

From the first image, an estimate of the repeatability of the phenomenon can be gained. The overlaid flame images are almost directly on top of each other in the near-field region indicating very strong phase locking of the vortical perturbations, and therefore the flame, to the excitation field. Farther downstream, more scatter can be seen...
in the flame position. As will be shown later, the effect of phase jitter is substantially stronger for the circular bluff body.

These images indicate that the flame sheet dynamics are controlled by their interaction with the convecting vorticity field, as opposed to a direct response to the velocity oscillations directly associated with the acoustic field. This illustrates that understanding this flame interaction problem requires an understanding of the growth, propagation, and decay of these vortices in a reacting, harmonically oscillating flow field.

![Figure 5. (LEFT) Dependence of instantaneous (circles) and ensemble averaged (squares) vorticity upon axial location, in the absence of acoustic excitation. (RIGHT) Dependence of ensemble averaged vorticity profile upon axial location at 45° phase (circles). The amplitude of excitation is \( u' / U_\infty = 0.29 \). Time averaged vorticity also shown for reference (squares). For both cases \( \phi = 0.71 \).](image)

We next consider the spatial evolution of the vorticity field at different phases of the cycle more quantitatively. This was accomplished by determining the maximum value of the vorticity at each axial location, \( x \). This presumably corresponds with the trajectory of the vortex sheet. As a baseline, Figure 5 (left) plots an instantaneous and time averaged value of this maximum vorticity value for the unforced case. The unforced case shows the intense vorticity at the bluff body separation point, which monotonically decays downstream, as discussed elsewhere in the literature\(^33,34\). Figure 5 (right) overlays this same time averaged value with the ensemble averaged value at one phase of the cycle. The forced case clearly shows the phase locked modulation of vorticity amplitude about this mean value. However, the convecting vortical structure is very rapidly dissipated; i.e., two clear structures are clearly evident, and a third and fourth ones of very small amplitude can be seen, albeit faintly.

In order to understand the processes controlling the evolution of these vortical structures, experiments were performed to compare the vortical dissipation rate for various temperature ratios across the flame, including the limiting case of non-reacting flow. Comparisons of the flow field with no heat release and significant amount of heat release requires care, however. As discussed in introduction (section I.C), in the non-reacting flow, both the wake mode and the shear layer mode are present. In contrast, for reacting flows with high dilatation ratios, only the shear layer mode is present. As such, in order to make a meaningful comparison, a splitter plate was introduced into the wake for the non-reacting studies, which suppresses the wake mode\(^3\), as is described by Anderson and Szewczyk\(^30\).

\(^3\) Comparisons of the non-reacting flow-fields with and without the splitter plate are detailed in the Appendix.
The effect of acoustic excitation on the ensemble averaged vorticity field with and without the flame is shown in Figure 6 and Figure 7. Figure 6 shows the vorticity variation along the upper shear layer, at the same phase of excitation with and without the flame, at identical excitation amplitudes. It is clear that there are both amplitude and phase differences between the vortical structure dynamics. Because these measurements are taken with respect to the excitation, not the acoustic velocity itself at the bluff body, it is not clear at this point what the source of this phase difference is (note that the presence of the flame causes acoustic wave reflection and, therefore, changes the phase of velocity perturbation). Figure 7 compares these same vortical responses, but at different phases, 180° and 45°, presented to allow for comparison when the structures are at approximately the same location, highlighted by the arrows indicating the peaks. (The phase locked data was not taken at sufficiently fine resolution to have results where the two are exactly on top of each other). It is clear that the vorticity fields are quite similar, just different in magnitude.

Figure 8. Dependence of vorticity fluctuations upon axial location for both reacting (green) and non-reacting (red) cases, for same amplitude of excitation \( u'/U_\infty = 0.29 \) but at nine different phases. The envelopes of these oscillations are illustrated in the figure on the right \( (\phi = 0.71) \) for reacting case.

These curves appear to indicate that the reacting case dissipates much more quickly, as we had anticipated, due to volume dilatation, baroclinic torque and higher viscous diffusion in the reacting case. However, because the mean level of vorticity is different for the two cases, the reality is actually more complex. In order to better visualize the vortical decay rate, the vorticity plots are overlaid for nine phases throughout the cycle at fixed excitation amplitude. In this way, the propagation and dissipation of the vortical structures is clearly evident, see Figure 8 (left).
envelope of the minima and maxima of these curves (Figure 8, right) then reveals the decay of the vorticity modulations.

![Figure 9. Dependence of the envelope of vorticity fluctuations upon axial locations, with (squares) and without combustion (circles). The amplitude of excitation in both cases is $u'/U_\infty = 0.29$. For reacting case, $\phi = 0.71$.](image)

Figure 9 plots the difference between these envelope curves. Clearly, there is a difference in the evolution of the vorticity, but the actual decay rate appears roughly comparable. The reduced amplitude of vorticity in the reacting case is apparently associated with the downstream translation of the field in the non-reacting case. As such, drawing definitive conclusions from this result is not possible. However, it may be that, even with the addition of the splitter plate, the differences between the two situations are too significant to draw a meaningful comparison; e.g., the necessarily different excitation velocities at the bluff body due to wave reflections from the flame, the influence of the splitter plate on the vortical field, and so forth. Furthermore, the effects of phase jitter, not significant within the first convective wavelength, but much more significant farther downstream, causes a reduction in ensemble averaged amplitude as well, an effect which we have only started to quantify, see discussion in next section.

![Figure 10. Dependence of vorticity fluctuations upon downstream location for five different equivalence ratios, $\phi = 0.63$ (stars), 0.65 (diamond), 0.68 (squares), 0.72 (circles) and 0.74 (x’s). Temperature ratios shown in graph correspond to calculated adiabatic flame temperatures. For all cases the amplitude of excitation is $u'/U_\infty = 0.29$ and the phase w.r.t driving is $\psi = 135^\circ$.](image)

An additional set of data were obtained at a single phase at five equivalence ratios, as seen in Figure 10. These correspond with five temperature ratios and, presumably, different vorticity dissipation rates. This graph shows that the vorticity fluctuations are nearly at the same phase, but monotonically decrease in magnitude with increases in equivalence ratio, or more fundamentally temperature ratio across the flame. This was the anticipated result, but does not allow for an assessment of the relative roles of diffusion, volume dilatation, and baroclinic torque. Though further studies are currently underway to better elucidate the relative significance of these processes, certain inferences can be drawn about the nature of the roles played by these processes. For this reason, consider Figure 11,
which plots instantaneous vorticity profiles and flame edges for these five equivalence/temperature ratios. Examination of these plots reveals that the vortices exist primarily in the high temperature, post-flame regions of the flowfield – this is the region of high diffusivity, and therefore rapid vorticity diffusion. As such, it should be expected that the vorticity decay rate should increase monotonically with temperature ratio. However, given the relatively modest temperature ratio changes, the differences seen in the instantaneous plots seem too substantial to explain purely from a diffusivity argument. For instance at the lowest temperature ratio, five distinct vortices are seen while only two can be seen in the highest temperature ratio – however, the diffusivity between these two cases differs only by a factor of 17%. Moreover, comparison of the ensemble averaged vorticity in Figure 10, which indicates similar trends, but substantially less significant than seen in the instantaneous pictures, illustrates that phase jitter phenomenon in the vortices farther downstream is also significant.

These images also suggest that volume dilatation does not play a significant role in the downstream evolution of these vortices, due to the fact that the vortex is always on the product side of the flame. Note that this volume dilatation mechanism is only present when the vortex passes through the flame. If the vortex remains always on one side or the other of the flame, the mechanism is not present. It may be that volume dilatation effects are significant in the very vicinity of the bluff body where the vortex straddles the flame in some of the images. However, the evolution of the vorticity farther downstream is apparently not strongly affected by dilatation.

Finally, we will argue in the next section that baroclinic effects, while evident in the data, are only of secondary importance.

$\phi = 0.63, \frac{T_b}{T_u} = 5.76$

$\phi = 0.65, \frac{T_b}{T_u} = 5.87$

$\phi = 0.68, \frac{T_b}{T_u} = 6.05$

$\phi = 0.72, \frac{T_b}{T_u} = 6.27$
In the figure, the vorticity field and the flame at different equivalence ratios are illustrated. For the same amplitude ($u'/U_\infty = 0.29$) and phase ($\psi = 135^\circ$) of acoustic excitation, each image plots the instantaneous vorticity field on which the flame position is overlaid. The equivalence ratio and temperature ratio are also indicated below each plot. The position of the bluff body is also shown for reference, drawn to scale. Dimensions on the axes are in meters.

**B. Comparison between Vorticity fields of Triangular and Circular Bluff-Bodies.**

The above results were obtained with the triangular bluff body (referred to as TBB, hereon) that employed a sharp separation point. In this section, we consider how these results are altered with a circular bluff body (referred to as CBB, hereon), where the separation point is not well-defined and can possibly exhibit stochastic characteristics. An overlay of instantaneous flame images and vorticity field are plotted in Figure 12 at three phases of the cycle. On an instantaneous basis, the results are quite similar to those from the prior section, with the exception of substantially more asymmetry in the upper and lower flame branches. This suggests that the vortex sheet dynamics of the top and bottom shear layers are not as phase synchronized as in the TBB. This point can particularly be seen in the top-left plot, which shows an overlay of 128 flame images and the ensemble averaged vorticity field. The flame position is clearly spread out substantially and the vorticity is smeared out. This shows that there is a significantly wider variation in the position of the vortex (spatial jitter), its vorticity and hence the flame that is wrapped around it.
Interestingly, however, the vorticity decay rate is substantially lower than for the TBB at the same temperature ratio, i.e., three distinct vortices can clearly be seen downstream. Part of this may be due to the relative location of the flame and vortex sheet. Note that in many of the images that the vortex centers are in the cold, reactant regime, where the diffusivity is substantially lower.

To quantify the effect of phase jitter and how it compares for the TBB and CBB, Figure 13 (left) plots the locations of the vortex core center at one phase of the cycle, at the same amplitude of acoustic excitation. The figure indicates that the variation in CBB vortex core center is approximately twice that of the TBB. In other words, for the CBB, the vortices occupy positions that are further spread from the mean, compared to the TBB.

Because of phase jitter, ensemble averaging smears out the vorticity field, as discussed above. As such, some care must be taken in interpreting vorticity decay rates and amplitudes for the acoustically forced case. That is, taking the maximum vorticity amplitude from the ensemble averaged field, as done in the TBB case in the prior section, leads to a different answer than if one takes the average of the maximum vorticity values. In the rest of this section, we consider both approaches for interpreting the data.
A comparison of the ensemble averaged and instantaneous vorticity fields between the TBB and CBB illustrate this point; see Figure 14 (left). The ensemble averaged result shows similar spatial behavior between the two bluff bodies, but a clear reduction in vorticity amplitude in the CBB. This is due to averaging across vortices with time varying locations, as can be seen by comparing these ensemble averaged results with representative instantaneous ones. It can be seen that the instantaneous vorticity values are actually larger for the CBB than the TBB, opposite the result suggested by the ensemble averaged curves.

To illustrate this further, Figure 13 (right), plots the magnitude of the maximum vorticity value for each of the 128 images obtained at one excitation phase. The curves are slightly offset due to the higher mean value of vorticity for the CBB case in the forced case. From this result, one can see substantially larger instantaneous values and fluctuations in vortex strength in the CBB case relative to the TBB. Furthermore, the offset in mean value of vorticity of the CBB relative to the TBB is opposite to that in the unforced case, see Figure 14 (right); i.e., in the unforced case, the TBB mean vorticity distribution is everywhere higher than that of the CBB. Naturally, the vorticity distribution at the separation point will be dependent on the radius of curvature of the bluff-body at the point of separation, causing larger mean vorticity levels for the TBB.

A more conclusive explanation of the higher instantaneous vorticity for the CBB relative to the TBB requires knowledge of the vorticity magnitude at the separation point. For instance if the vorticity at the separation point for the CBB is only slightly lower than that for the TBB, then the fact that the vortex propagates mostly along the unburnt side of the flame for the CBB (Figure 12) as opposed to the burned gas for the TBB (Figure 4) could explain why the vorticity magnitudes are higher for the CBB downstream.

C. Excitation Amplitude and Baroclinic Effects

Experiments were performed over a range of amplitudes in order to understand nonlinear effects. That is, as the vortex strength increases, it causes increased corrugation of the flame front. Further, it complicates the baroclinic vorticity term which depends upon the relative orientation of the pressure and density gradients. For example, see Sinibaldi et al.\textsuperscript{26} and Bray and Louch\textsuperscript{28} for a discussion of the change in dynamics of flame-vortex interactions for a nominally planar flame with changing vortex strength.
Figure 15. (LEFT) Velocity-vorticity amplitude transfer function. Case A: CBB, maximum of ensemble averaged amplitude, Case B: TBB, maximum of ensemble averaged amplitude, Case C: TBB, average of maximum amplitude, Case D: CBB, average of maximum amplitude (RIGHT) Dependence of spatial jitter in vortex core location on amplitude of excitation. For both cases, $\phi = 0.71$

Because of phase jitter effects, we parameterize the vorticity evolution by the average of its peak value from the 128 images, rather than maximum of its ensemble average. The resulting velocity-vorticity response is illustrated in Figure 15 (left, cases C & D). For the range of amplitudes at which excitation was carried out, the response is apparently quite linear. Increasing excitation lowers the amount of jitter in vorticity location, as is shown in Figure 15 (right), apparently at comparable rates. The linearity in the vorticity-velocity response function can also be seen for the TBB flow-field if the ensemble averaged vorticity amplitude is used Figure 15 (left, case B). A distinct difference is seen for the CBB flow-fields where phase jitter leads to a non-discernible trend. The effect of jitter is also seen in the TBB flow-field, by the difference between cases B and C.

Figure 16: Illustration of vorticity production by the baroclinic mechanism. The figure shows instantaneous vorticity field and flame edge at excitation amplitude $u'/U_\infty = 0.29$ and phase $= 45^\circ$. The equivalence ratio is $\phi = 0.71$. Directions of pressure gradient,$\nabla p$ (green arrows) and density gradient,$\nabla \rho$ (crimson arrows) are indicated. The curved arrows indicate the direction of circulation of the vortex.
Figure 17: Illustration of baroclinic vorticity production. (LEFT) Variation in vorticity (ensemble averaged) in a direction transverse to the flow, measured at \( x/D = 0.5 \). Regions of flow where vorticity is produced by the baroclinic mechanism are encircled. The vorticity levels plotted for different cases are normalized by the maximum within each case. (RIGHT) Dependence of baroclinic induced vorticity magnitude upon amplitude of excitation.

The effect of baroclinic torque grows in significance with disturbance amplitude. The region of the flow where baroclinic torque is produced can be seen in Figure 16, where positive vorticity can be seen along the flame front in regions of predominantly negative, shear induced vorticity and vice versa (see circled regions). Assuming that the pressure gradient is favorable, then this would be the expected trend for the given flame inclination angle with respect to the mean flow direction. As such, the maximum baroclinic torque will be produced if the flame is curved in a way such that its tangent is aligned in the direction of the flow; i.e., at the top and bottom points of the flame that is wrapped around the vortex. The presence of this baroclinically induced vorticity can also be seen in many of the prior images shown in this paper.

Clearly, this effect will grow in significance with amplitude of excitation, as it causes a corresponding increase in flame corrugation. This can be seen from, which plots the ensemble averaged transverse vorticity profile at an axial location of \( x/D = 0.5 \). The vorticity distribution is normalized by the maximum vorticity to emphasize changes in shape of this profile. At low amplitudes of excitation, the vorticity profile on each side of the flame has a single hump, due to shear induced vorticity. As the amplitude grows, new features can be seen developing above and below this hump. First, an oppositely signed region of vorticity grows on the outer edge of the shear layer. Second, the vorticity profile on the inner side is distorted. These features monotonically grow in magnitude with amplitude of excitation. This is quantified in Figure 17 (right), which plots the magnitude of the oppositely signed hump on the outside edge of the shear layer, showing its nonlinear amplitude dependence upon excitation amplitude.

Nonetheless, though baroclinic effects grow in prominence with disturbance amplitude, it appears that their effects are still secondary relative to shear generated vorticity, at least for the disturbance amplitudes considered here. This can be seen from the size of the baroclinic induced secondary peak in the outer shear layer relative to the main, shear induced peak in, as well as from comparisons of the baroclinic-induced vorticity magnitudes plotted in Figure 17 (right) relative to the other representative vorticity values reported in this paper.

IV. Concluding Remarks

The key result of this paper is quantification of the downstream evolution of the vorticity field that is perturbing the flame. These results suggest that the decay of vortical structures downstream is controlled by viscous diffusion, except possibly in the region near the separation point where volume dilatation effects may also be significant. Baroclinic effects grow nonlinearly with amplitude but are apparently still negligible relative to the shear generated vorticity, at least for the excitation amplitudes considered here.

However, the effects of phase jitter also play an important role in the decay of ensemble averaged vorticity. Some results were shown here illustrating this point, but significant further work is needed to better quantify this effect, even for the TBB. In addition, further analysis of our data is needed to quantify the downstream evolution of the total circulation of each vortex core (not influenced by diffusion), as well as the vortex core size.

Two further sets of studies are needed to conclude this work (1) an understanding of the factors controlling the initial vorticity magnitude and vortex size and (2) quantification of the vorticity dissipation rate downstream.
Further analysis of our data as well as additional measurements are planned to develop scaling/prediction capabilities for these quantities which will allow for an assessment of vorticity evolution at the flame. In turn, this will allow for a better prediction capability for the flame’s heat release response, as the velocity field locally perturbing the flame is what is ultimately responsible for these fluctuations.

Appendix

This appendix compares the non-reacting flow field with and without the splitter plate. Figure 18 shows ensemble averaged streamlines without the splitter plate, where four small vortices, symmetrically placed about the bluff body centerline are evident. Further downstream, a larger vortex, due to the wake mode is also evident. Figure 19 shows the corresponding flow field with the splitter plate. The presence of three vortices in the shear layer is clearly evident, but the larger vortex present in the prior case is absent.

![Figure 18: Illustration of streamlines and velocity vectors for non-reacting flow with acoustic excitation.](image1)

![Figure 19: Illustration of streamlines and velocity vectors for acoustically excited non-reacting flow with a splitter plate in the wake. The excitation amplitude is \( u'/U_\infty = 0.29 \).](image2)

References