Nonlinear Control Design for Propulsion Systems

by

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Outline

- Research Accomplishments
  - Nonlinear Robust Controller Synthesis Framework
  - Hierarchical Nonlinear Switching Control

- Current Research Directions
  - Minimal Complexity Control
  - Nonlinear Hybrid Control

- Ongoing Research
Archival Research Publications

- Journal publications: 16
  - 6 appeared, 4 accepted, 6 submitted

- Conference publications: 22
  - 20 appeared, 2 accepted

- Conferences
  - American Control Conference
  - Conference on Control Applications
  - IEEE Conference on Decision and Control
Aerodynamic Instabilities of Rotating Stall and Surge

- Rotating Stall and surge
  - Compression system oscillations can damage jet engine components and cause flame-out to occur
Rotating Stall and Surge Instabilities

- High pressure rise $\Rightarrow$ reduce stages (dimensions, weight)

- **Stall**: Drastic loss of pressure rise in compressor; hysteresis

- **Surge**: Global compression system oscillations; limit cycle
  - Axisymmetric equilibria
  - Stall equilibria
  - Stable and unstable branches
Research Accomplishments

- Nonlinear controller synthesis for nonlinear systems
  - Based on inverse optimality
  - Disturbance rejection properties
  - **Guaranteed** robustness
  - Actuator saturation
  - Adaptive control
  - Global stability guarantees
Robust Control for Propulsion Systems

Unified Robust Optimal Nonlinear Control Framework

- Robustness, disturbance rejection, control saturation
Robust Control for Rotating Stall and Surge

- Squared stall amplitude versus time and pressure versus time

![Graphs showing squared stall amplitude and compressor pressure over time with robust and backstepping controllers.](image-url)
Adaptive Control Framework

- Parameter update law
  - Identify unknown system parameters
  - Adjust feedback gains to account for system variation

- Parametric robustness, disturbance rejection, robustness to unmodeled dynamics
Modeling Research Accomplishments

- Generalized multi-mode nonlinear state space model for rotating stall and surge in axial flow compressors
  - Novel nonlinear control strategy accounting for the interaction between higher-order system harmonics
- Developed a lumped parameter model for surge in centrifugal flow compression systems
  - Account for spool speed transients
  - Nonlinear control to avoid surge instabilities
  - Account for control amplitude and rate saturation constraints
Multi-Mode Control

- The first and higher-order velocity field harmonics are strongly coupled during stall inception
- Not all nonlinear phenomena (e.g., bifurcations, limit cycles, hysteresis, etc...) can be captured with a low order model
- Difficulties in applying bifurcation-based and backstepping controllers to high-order models
  - Bifurcation diagrams involve more complicated phenomena (e.g., fold and Hopf branches)
  - High-order subsystem dynamics
- A control strategy for multi-mode models is needed
Backstepping Controller Design

- **Globally** stabilizing backstepping controller for a one-mode model given by Krstić et al., 1995

- In the two-mode case the backstepping controller drives the system to a stalled equilibrium
Axial Compression System

- Quasi-steady, axisymmetric compressor characteristic map

\[ \psi_c(\Phi) = \psi_{c0} + 1 + \frac{3}{2} \Phi - \frac{1}{2} \Phi^3 \]

- \( \Phi \) is the circumferential averaged flow

- \( \Psi \) is the pressure rise
**Observations**

- The first and higher-order velocity field harmonics strongly interact during compressor stall inception
  - Higher order modes must be accounted for in the control design processes

- Nonlinear multi-mode *state space* model:
  - Cannot be represented in strict feedback form
  - Is not feedback linearizable
  - HJB solutions are intractable
  - The linearization is unstabilizable
Hierarchical Control Architecture
Hierarchical Switching Control Strategy

- Assume that $u(t) = \phi_\lambda(x(t))$ stabilizes $x = x_\lambda$ for $x_0 \in \mathcal{D}_\lambda$

- How can we choose the operating conditions needed for switching?
- How can we define an effective switching strategy?
- How can we guarantee that homotopic switching works?
Hierarchical Feedback Control Strategy

- Assumed that $u(t) = \phi_\lambda(x(t))$ stabilizes $x = x_\lambda$ for $x_0 \in \mathcal{D}_\lambda$

- It follows that $u(t) = \phi_{\lambda_S(x(t))}(x(t))$ stabilizes $x=0$ for $x_0 \in \mathcal{D}_c$

- $p(\lambda_S(x(t)))$ is a nonincreasing lower semicontinuous function

- There exists $\{t_k\}_{k=0}^\infty$ such that $p(\lambda_S(x(t_{k+1}))) < p(\lambda_S(x(t_k)))$
Hierarchical Control Design

- Hierarchical Switching Control

- Provides theoretical foundation for designing gain scheduled controllers

- Guarantees stability over a wide range of system operating conditions

- Global stability, robustness, state and control constraints
Hierarchical Nonlinear Switching Controller

- The hierarchical switching controller guarantees stability for an arbitrary number of modes

- Two-mode case

- Throttle opening versus time
Amplitude and Rate Saturation Control

- Framework characterizes the fastest admissible rate of change of $\lambda(t)$

- $\lambda(t)$ is proportional to the throttle opening
  - Constraining the dynamics of $\lambda(t)$ places a rate constraint on the throttle opening

- **Amplitude** saturation constraints can be enforced by choosing $\lambda_{\text{max}} < 1$ such that

$$D_{\text{max}} \triangleq \bigcup_{0 \leq \lambda \leq \lambda_{\text{max}}} D_{\lambda} \subseteq \left\{ \text{region where the system is constrained to operate} \right\}$$
Rate Saturation Control Implementation

- Two-mode case with rate saturation constraint $|\dot{\gamma}_{th}| \leq 1$
Robust Stabilization of Axial Flow Compressors

Robustness Extensions

- Compression system uncertainties
  - Parametric uncertainties
  - Unmodeled system dynamics
  - System delays

- Develop a robust control strategy for multi-mode axial flow compressors

- The compressor map is uncertain
  - Modeling errors
  - In-service changes due to aging
    \[ \psi_C(\phi) = \psi_C^{\text{nom}}(\phi) + \Delta \psi(\phi) \]
Limitations of Proposed Control Approaches

- Full state feedback architecture - MG3, A, Φ, and Ψ
- Optimality with respect to a derived cost functional as opposed to a designer specified cost functional
- Static (memoryless) controllers
  - Constant gain multiplication of measurements
  - Possesses gain at all frequencies (has no rolloff)
Current Research

- Fixed-architecture control
  - Output feedback dynamic control
  - Sensor architecture flexibility
- Compressor performance versus
  - Direct optimality
- Controller complexity reduction
  - Controller architecture
  - Controller order

\{ sensor accuracy
  processor throughput
  actuation levels
  disturbance rejection
  modeling accuracy \}
Controller Architectures

- Bifurcation-based control
  
  \[ \gamma_{\text{throttle}}(A) = \gamma_0 + kA^2; \quad \gamma_{\text{throttle}}(A, \dot{\Phi}) = \gamma_0 + k_1A^2 - k_2\dot{\Phi} \]

  \(-A\) is the amplitude of rotating stall, \(\dot{\Phi} = \frac{d}{dt}(\text{compressor flow})\)

- Changes bifurcation structure of controlled system at onset of rotating stall

  \(-\) Hard subcritical bifurcation changed to soft supercritical bifurcation

- Ineffective for surge

- Demanding two-dimensional sensing requirements
Two-Dimensional Sensing

- Circumferentially distributed pressure sensor arrays around compressor annulus

- DFT software for spatial and temporal filtering for computing the first circumferential spatial harmonic of rotating stall
Minimal Complexity Control Law Synthesis

- Require that $\tilde{G}$ satisfies an absolute stability criterion

- Perform fixed-structure optimization over dynamic controller gains
  - Fixed-order dynamic compensators; pressure feedback only!

- Closed-loop stability and disturbance rejection guaranteed
Phase Portrait of Pressure versus Flow

- Undisturbed case
  - IC’s at angular intervals of $\frac{\pi}{6}$ about the maximum pressure operating point
Stall Cell Amplitude versus Time

- Disturbed case

![Graph showing Stall Cell Amplitude versus Time](image)

- Open-Loop
- Disturbance Rejection
- Liaw and Abed
- Badmus et al.
Pressure Rise in Compressor versus Time

- Disturbed case
Throttle Opening versus Time

- Disturbed case

![Graph showing throttle opening versus time with different lines representing Open-Loop, Disturbance Rejection, Liaw and Abed, and Badmus et al.](image)
Nonlinear Hybrid Control

- Energy-based resetting feedback control
  - Exploit energy flow and power ideas in feedback systems

- Impulsive differential equations
  - Stability, dissipativity, feedback interconnections, optimality
Reseting Control Design

- Plant dynamics are continuous-time
- Controller dynamics are hybrid

\[-(t, x_c(t), y(t)) \in S_c\]
\[-S_c = \mathcal{T} \times \mathbb{R}^n, \quad S_c = [0, \infty) \times \mathcal{Z}\]

- Only the controller states are reset
- Plant energy \(V_p(x_p) > 0\), emulated controller energy \(V_c(x_c) > 0\), total energy \(V(x) \triangleq V_p(x_p) + V_c(x_c)\)

- IDEA: When the controller accumulates emulated energy, reset the controller states, so that energy cannot return to the plant
One-Way Resetting Controller

- Define the resetting set $\mathcal{S}$ as

$$
\mathcal{S} \triangleq \left\{ x = \begin{bmatrix} x_p \\ x_c \end{bmatrix} : \text{Plant energy} \geq 0 \right\}
$$

- If $x \not\in \mathcal{S}$ then the plant energy is decreasing

- If $x \in \mathcal{S}$ then the controller states are reset, and $x$ leaves $\mathcal{S}$

- Controller effects a one-way energy flow

- Knowledge of $x_c$ and $y$ is sufficient to determine whether or not $x \in \mathcal{S}$
Active Control of Combustion Instabilities

\[
\frac{\partial \rho}{\partial t} + v_g \cdot \nabla p = \mathcal{W}
\]
\[
\rho \frac{\partial v_g}{\partial t} + \rho v_g \cdot \nabla v_g + \nabla p = \mathcal{F}
\]
\[
\frac{\partial p}{\partial t} + \gamma p \nabla \cdot v_g + v_g \cdot \nabla p = \mathcal{P}
\]

- Coupling of linear acoustics and nonlinear heat release
- Two-mode, time-averaged combustion model (Culick’s model)
Uncontrolled Combustion System

- Phase portrait and state response
Time-Dependent Resetting Controllers

- Output versus time
- Plant, controller, and total energy
State-Dependent Resetting Controllers

- Output versus time:
  Finite-time stabilization guaranteed

- Plant, controller, and total energy
Ongoing Research

- Controller implementation on GT’s axial flow compressor rig
  - Switching controller
  - Pressure feedback controller
- Energy-based resetting feedback control
  - Robustness guarantees
  - Output feedback controllers
- Digital (sampled-data) control
  - Hybrid systems
Technology Transitions

- GE Corporate Research and Development, Schenectady, NY
  - S. MacMinn
  - 3/17/99

- United Technologies research Center, Hartford, CT
  - A. Banaszuk, C. Jacobson
  - 6/17/99