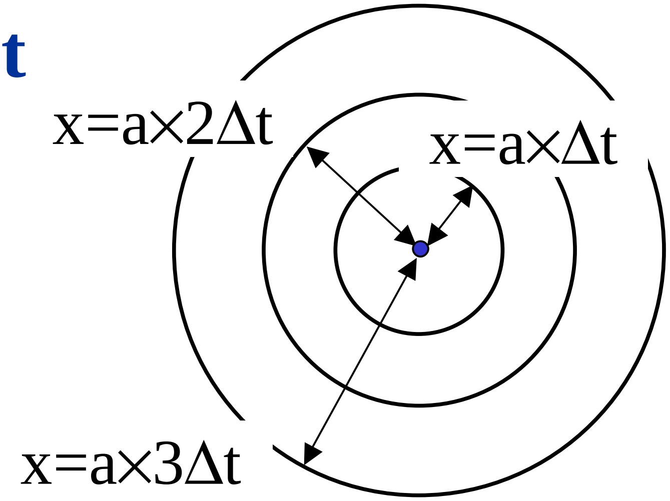
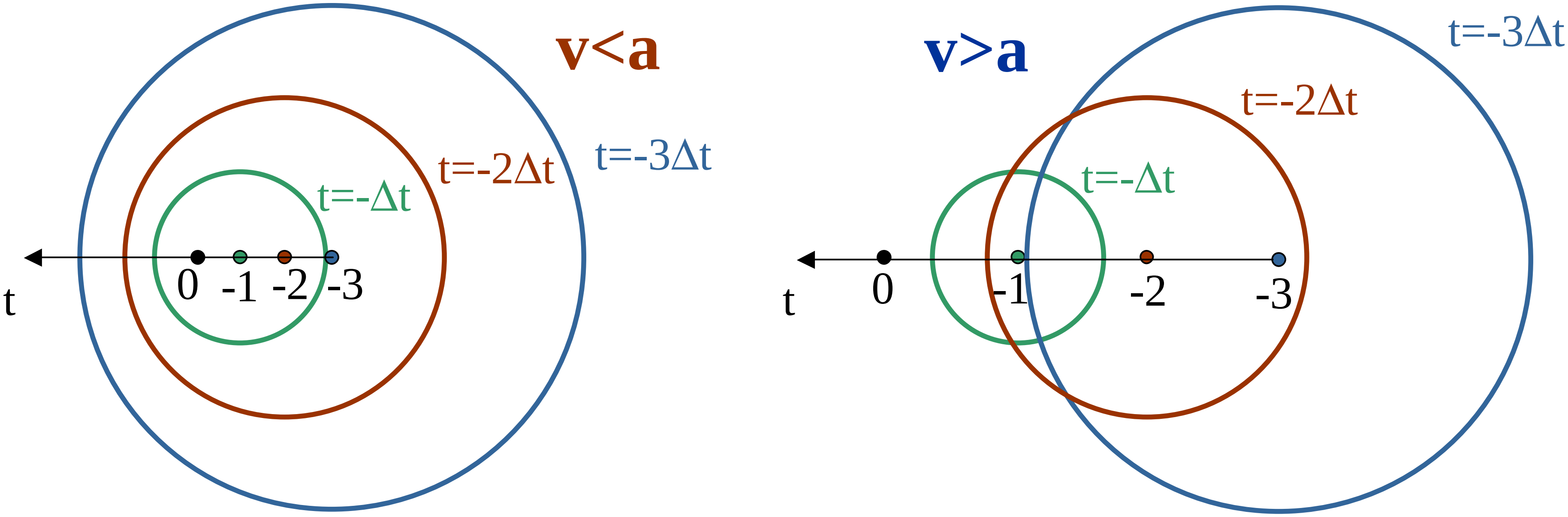
Mach Angle and Mach Number

- Looking for relationship between speed of sound and flow speed (or speed of body moving through fluid)
- Consider small body (point) moving in stagnant fluid
  - continuously "launches" weak pressure disturbances (e.g., from "pushing" fluid)
- Disturbances travel outward spherically at sound speed (a)
- **Look at disturbances generated at equally spaced time intervals**
- Start with body moving with **v<<a**
  - e.g., nearly stationary or moving through incompressible liquid



Subsonic and Supersonic Motion

- Now compare two bodies, one moving with **v<a**, **subsonic** other body moving with **v>a**, **supersonic**

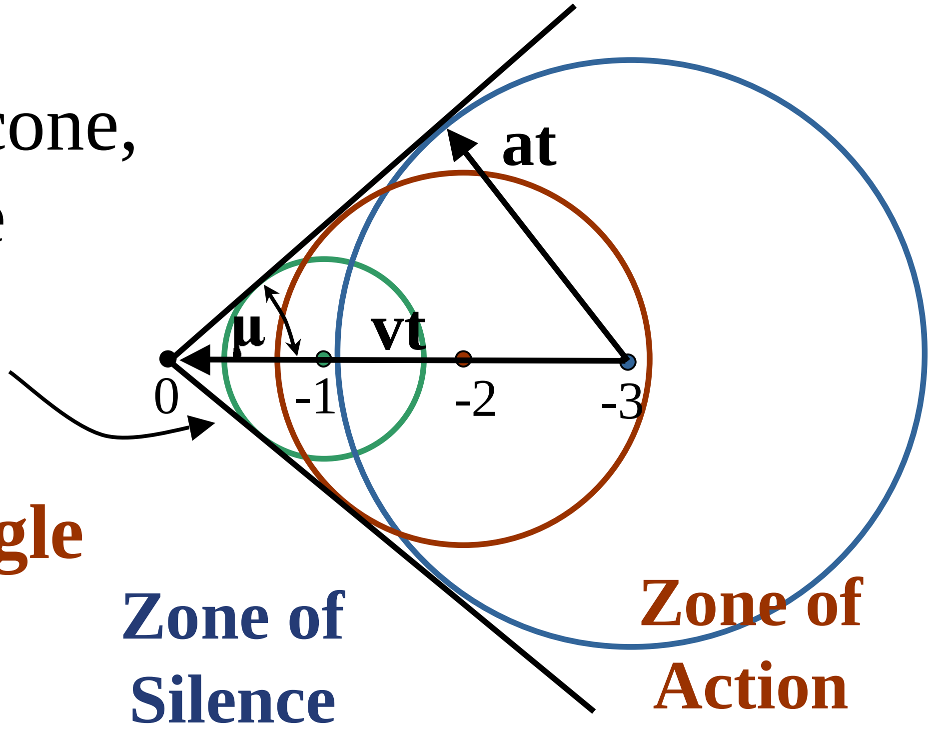


- Subsonic body always behind sound waves launched from previous positions
- Supersonic body moves ahead of previous sound waves

Mach Wave and Mach Angle

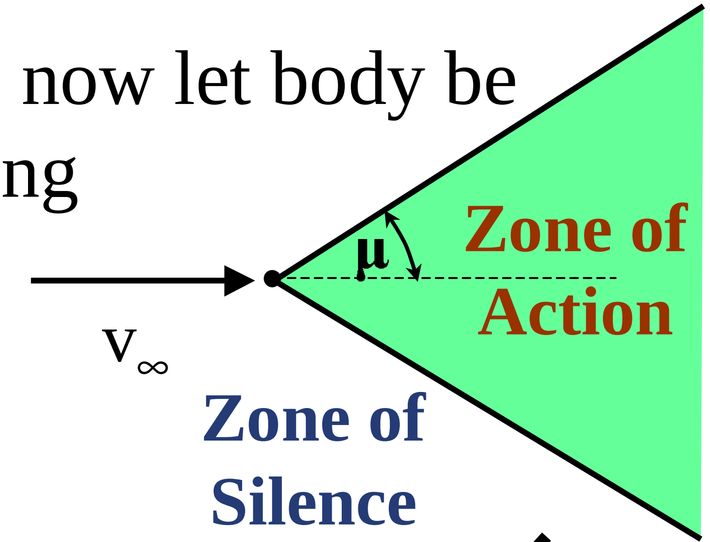
- For supersonic flow, can define region where disturbance has had an effect (been “heard”)
- Conical region delineated by tangents to sound wave spheres
- Waves coalesce at edge of cone, produce largest disturbance
  - **Mach wave (Mach line)**
- Angle between Mach line and body motion, **Mach angle**

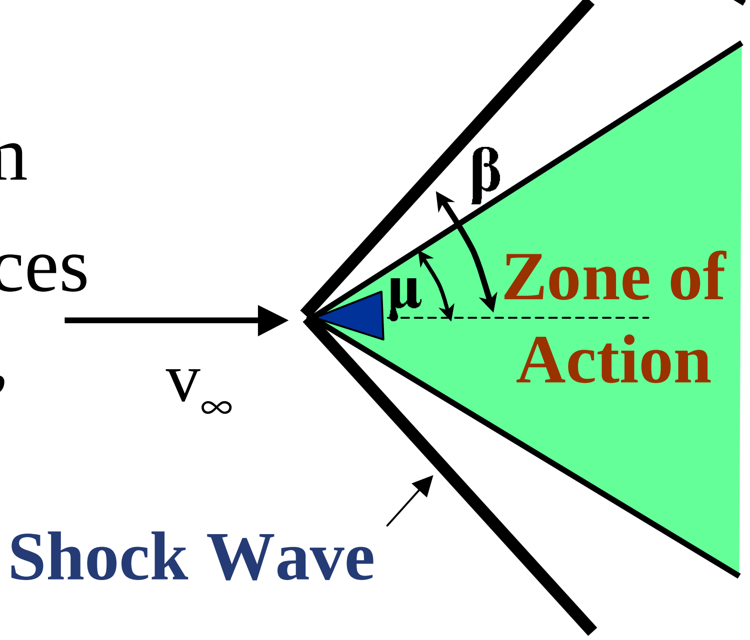
$$\mu = \sin^{-1}\left(\frac{at}{vt}\right) = \sin^{-1}\left(\frac{a}{v}\right)$$



Mach Cone and Shock Waves

- Change frame of reference, now let body be stationary and flow is moving
- Weak disturbances from presence of body
  - can only be felt inside Mach cone
  - can not be felt upstream
- Strong pressure disturbances (nonisentropic) can occur, they coalesce to form **shock waves** $\beta \geq \mu$





Mach Number

- So flow/body speed relative to sound speed is fundamental ratio
- **Mach number** $$M \equiv \frac{v}{a}$$ **(VI.3)**

named for Ernst Mach,
Austrian ~1870

- In terms of Mach number, the **Mach angle** is

$$\mu = \sin^{-1}\left(\frac{1}{M}\right)$$ **(VI.4)**

- Loose demarkation of flow regimes ($M_\infty = v_\infty / a_\infty$)

$M_\infty < 0.3$	**"incompressible"**	$\Delta\rho < 5\%$ effect
$0.3 < M_\infty < 0.8$	**subsonic**	ρ changes with v
$0.8 < M_\infty < 1.2$	**transonic**	shocks for M>1
$1.2 < M_\infty < 3$	**supersonic**	stronger ρ changes
$3 < M_\infty$	**hypersonic**	very strong shocks

Adiabatic Flow Ellipse

- Another way to look at M effects
- Energy equation

adiabatic stream tube

$$h_o = h + \frac{v^2}{2} = \text{const}$$

- Stagnation T_o also constant (thermally & calor. p.g.)

$$T_o = T + \frac{\gamma-1}{2}\frac{v^2}{\gamma R} = \text{const}$$

$$\frac{2}{\gamma-1}\gamma RT + v^2 = \text{const}$$

$$\frac{2}{\gamma-1}a^2 + v^2 = v_{max}^2 = \frac{2}{\gamma-1}a_o^2$$

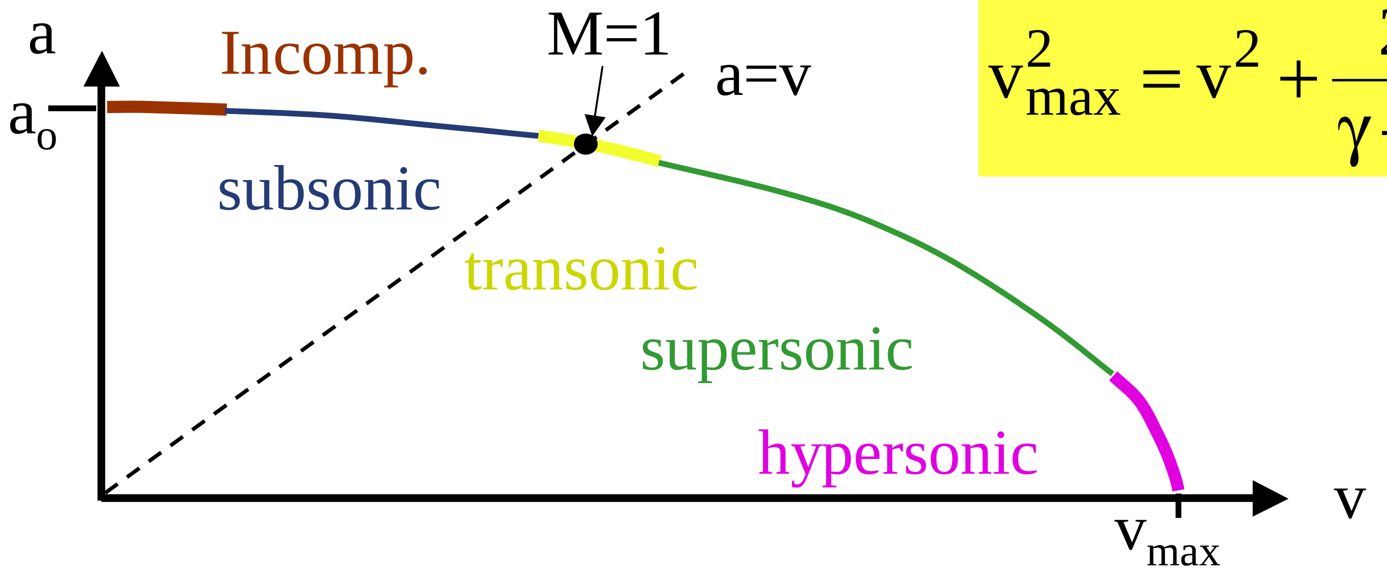
(VI.5)

Stagnation speed of sound
(no kinetic energy left, v=0)

Maximum velocity possible
(no thermal energy left, T=0)

Adiabatic Flow Ellipse (con’t)

- Transition from low speed (a_o) to high speed (v_{max})



$$v_{max}^2 = v^2 + \frac{2}{\gamma-1}a^2 = \frac{2}{\gamma-1}a_o^2$$

incomp.	$v \ll a$, $da \ll dv$, little change in a(T)
subsonic	$v \lesssim a$, M changes primarily to changes in v
transonic	$\lvert v-a \rvert \ll v,a$
supersonic	$v > a$, M changes through substantial changes in v and a(T)
hypersonic	$v \gg a$, $dv \ll da$, M change mostly due to a(T) changes