Resistance Strain Gage Circuits

- How do you measure strain in an electrical resistance strain gage using electronic instrumentation?

- Topics
  - basic gage characteristics
  - Wheatstone Bridge circuitry
  - Bridge completion, balancing, calibration, switching

- Course Notes provide complete coverage.

- See reference text for further details.
Resistance in Metallic Conductor

Resistance Equation:

\[ R = \rho \frac{L}{A} \]

Change in R:

\[ \Delta R = \rho \frac{L}{A} \left( \frac{L + \Delta L}{A + \Delta A} - 1 \right) \]

Differential change in R:

\[ \Delta R \equiv \frac{dR}{R} = d \left( \rho \frac{L}{A} \right) \]

or:

\[ \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \]

After a bit of manipulation:

\[ \frac{dR}{R} = \frac{d\rho}{\rho} + (1 + 2\nu) \varepsilon \]

DEFINE Gage Factor (GF):

\[ GAGE \ FACTOR = GF = \frac{dR / R}{\varepsilon} \]

GF is the slope of these curves.

Wire before and after strain is applied.
Examples

Table 1. Gage Factors for Various Grid Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Gage Factor (GF)</th>
<th>Ultimate Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Strain</td>
<td>High Strain</td>
</tr>
<tr>
<td>Copper</td>
<td>2.6</td>
<td>2.2</td>
</tr>
<tr>
<td>Constantan*</td>
<td>2.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Nickel</td>
<td>-12</td>
<td>2.7</td>
</tr>
<tr>
<td>Platinum</td>
<td>6.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Silver</td>
<td>2.9</td>
<td>2.4</td>
</tr>
<tr>
<td>40% gold/palladium</td>
<td>0.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Semiconductor**</td>
<td>~100</td>
<td>~600</td>
</tr>
</tbody>
</table>

* similar to “Ferry” and “Advance” and “Copel” alloys.
** semiconductor gage factors depend highly on the level and kind of doping used.

Example 1

Assume a gage with GF = 2.0 and resistance 120 Ohms. It is subjected to a strain of 5 microstrain (equivalent to about 50 psi in aluminum). Then

$$\Delta R = GF \varepsilon R$$

$$= 2(5e - 6)(120)$$

$$= 0.0012 \text{ Ohms}$$

$$= 0.001\% \text{ change!}$$

Example 2

Now assume the same gage is subjected to 5000 microstrain or about 50,000 psi in aluminum:

$$\Delta R = GF \varepsilon R$$

$$= 2(5000e - 6)(120)$$

$$= 1.2 \text{ Ohms}$$

$$= 1\% \text{ change}$$
Resistance Measuring Circuits

Current Injection

\[ \text{Constant Current Source} \]

\[ i \quad R 

\]

Ballast Circuit

\[ E \quad R_b \quad R_g \]

Output:

\[ e = \frac{R_s}{R_s + R_g} \]

Small changes:

\[ de = \left[ \frac{dR_g}{R_s + R_g} - \frac{R_s dR_g}{(R_s + R_g)^2} \right] E \]

\[ = \frac{R_b R_s E}{(R_b + R_g)^2} \frac{dR_g}{R_g} \]

\[ = \frac{R_b R_s E}{(R_b + R_g)^2} GF \varepsilon \]

Optimal output \((R_b = R_g)\):

\[ de = \frac{GF}{4} \varepsilon E \]

\[ e + de = E/2 + GF/4 \varepsilon E \]

Impractical resolution problems

Output includes \(E/2\) plus an incremental, \(de\), which is VERY small!
Wheatstone Bridge Circuit

Output:
\[ e = \left[ \frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right] E = \frac{R_2 R_4 - R_1 R_3}{(R_1 + R_2)(R_3 + R_4)} E \]

Balance Condition:
\[ R_2 R_4 = R_1 R_3 \]
Differential output:

\[ de = \left[ \frac{R_1R_2}{(R_1 + R_2)^2} \left( \frac{dR_1}{R_1} - \frac{dR_2}{R_2} \right) + \frac{R_3R_4}{(R_3 + R_4)^2} \left( \frac{dR_3}{R_3} - \frac{dR_4}{R_4} \right) \right] E \]

Assuming initially balanced bridge:

\[ de = \frac{1}{4} \left[ \frac{dR_1}{R_1} - \frac{dR_2}{R_2} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4} \right] E \]

Using definition of GF and \( e + de = de \):

\[ e = \frac{GF}{4} \left[ \varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 \right] E \]

- The equation identifies the first order (differential) effects only, and so this is the “linearized” form. It is valid only for small (infinitesimal) resistance changes. Large resistance changes produce nonlinear effects and these are shown in Figure 3 where finite changes in \( R (\Delta R) \) in a single arm are considered for an initially balanced bridge.
- Output is directly proportional to the excitation voltage and to the Gage Factor. Increasing either will improve measurement sensitivity.
- Equal strain in gages in adjacent arms in the circuit produce no output. Equal strain in all gages produces no output either.
- Fixed resistors rather than strain gages may be used as bridge arms. In this case the strain contribution is zero and the element is referred to as a “dummy” element or gage.
Temperature Effects

- Gage material can respond as much to temperature as to strain:

<table>
<thead>
<tr>
<th>Material</th>
<th>Composition</th>
<th>Use</th>
<th>GF</th>
<th>Resistivity (Ohm/mil-ft)</th>
<th>Temp. Coef. of Resistance (ppm/F)</th>
<th>Temp. Coef. of Expansion (ppm/F)</th>
<th>Max Operating Temp. (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constantan</td>
<td>45% Ni, 55% Cu</td>
<td>Strain Gage</td>
<td>2.0</td>
<td>290</td>
<td>6</td>
<td>8</td>
<td>900</td>
</tr>
<tr>
<td>Isoelastic</td>
<td>36% Ni, 8% Cr, 0.5% Mo, 55.5% Fe</td>
<td>Strain gage (dynamic)</td>
<td>3.5</td>
<td>680</td>
<td>260</td>
<td></td>
<td>800</td>
</tr>
<tr>
<td>Manganin</td>
<td>84% Cu, 12% Mn, 4% Ni</td>
<td>Strain gage (shock)</td>
<td>0.5</td>
<td>260</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nichrome</td>
<td>80% Ni, 20% Cu</td>
<td>Thermometer</td>
<td>2.0</td>
<td>640</td>
<td>220</td>
<td>5</td>
<td>2000</td>
</tr>
<tr>
<td>Iridium-Platinum</td>
<td>95% Pt, 5% Ir</td>
<td>Thermometer</td>
<td>5.1</td>
<td>135</td>
<td>700</td>
<td>5</td>
<td>2000</td>
</tr>
<tr>
<td>Monel</td>
<td>67% Ni, 33% Cu</td>
<td></td>
<td>1.9</td>
<td>240</td>
<td>1100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nickel</td>
<td>-12</td>
<td></td>
<td>-12</td>
<td>45</td>
<td>2400</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Karma</td>
<td>74% Ni, 20% Cr, 3% Al, 3% Fe</td>
<td>Strain Gage (hi temp)</td>
<td>2.4</td>
<td>800</td>
<td>10</td>
<td></td>
<td>1500</td>
</tr>
</tbody>
</table>

- For “isoelastic” material:

\[
\varepsilon = \frac{dR}{R} \frac{1}{GF} = \frac{260}{3.5} = 74 \text{ microstrain/}^0 F
\]
Temperature Effects - cont’d

• Strains can appear due to differential expansion also:
  – Consider a “constantan” gage on aluminum
  – Constantan=8 \( \mu \text{strain} / F \) and aluminum=13 \( \mu \text{strain} / F \)

\[ \varepsilon = 13 - 8 = 5 \text{ microstrain} / ^0 F \]

• Combining with direct temperature effects in constantan:

\[ \varepsilon = 3 + 5 = 8 \text{ microstrain} / ^0 F \]

• Gages are cold-worked to match thermal properties of selected substrate materials:

<table>
<thead>
<tr>
<th>PPM/°F</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Molybdenum</td>
</tr>
<tr>
<td>6</td>
<td>Steel; titanium</td>
</tr>
<tr>
<td>9</td>
<td>Stainless steel, Copper</td>
</tr>
<tr>
<td>13</td>
<td>Aluminum</td>
</tr>
<tr>
<td>15</td>
<td>Magnesium</td>
</tr>
<tr>
<td>40</td>
<td>Plastics</td>
</tr>
</tbody>
</table>

Table 3. Materials for which Strain Gages can be Compensated (typical)
Gage Heating: Excitation Voltage Limits

- Gage output can be increased by increasing excitation
  - *this also increases current flow through gage*
  - *creates direct resistance heating (P=E²/R)*

Table 4. Strain Gage Power Densities for Specified Accuracies on Different Substrates

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Substrate Conductivity</th>
<th>Substrate Thickness</th>
<th>Power Density (r) (Watts/sq. in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Good</td>
<td>thick</td>
<td>2-5</td>
</tr>
<tr>
<td>High</td>
<td>Good</td>
<td>thin</td>
<td>1-2</td>
</tr>
<tr>
<td>High</td>
<td>Poor</td>
<td>thick</td>
<td>0.5-1</td>
</tr>
<tr>
<td>High</td>
<td>Poor</td>
<td>thin</td>
<td>0.05-0.2</td>
</tr>
<tr>
<td>Average</td>
<td>Good</td>
<td>thick</td>
<td>5-10</td>
</tr>
<tr>
<td>Average</td>
<td>Good</td>
<td>thin</td>
<td>2-5</td>
</tr>
<tr>
<td>Average</td>
<td>Poor</td>
<td>thick</td>
<td>1-2</td>
</tr>
<tr>
<td>Average</td>
<td>Poor</td>
<td>thin</td>
<td>0.1-0.5</td>
</tr>
</tbody>
</table>

Table Notes:
- Good = aluminum, copper
- Poor = steels, plastics, fiberglass
- Thick = thickness greater than gage element length
- Thin = thickness less than gage element length
Computations

- We can compute excitation limits...
- Power dissipated in a gage:
  \[ P_g = \frac{E^2}{4 R_g} \]
- Power density in the gage (power per unit area) from previous:
  \[ \rho = \frac{P_g}{A_g} = \frac{P_g}{L_g W_g} \]
- Maximum excitation depends on:
  - maximum power density allowed
  - gage area
  - gage resistance

\[ E_{\text{max}} = 2\sqrt{\rho L_g W_g R_g} \]
Wheatstone Bridge Compensation

- A desirable characteristic of the Wheatstone Bridge is its ability to compensate for certain kinds of problems.
- Consider temperature induced changes in strain: \( \varepsilon_t = \varepsilon_i + \varepsilon_i^T \):

\[
e = \frac{GF}{4} \left[ \varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 \right] E + \frac{GF}{4} \left[ \varepsilon_1^T - \varepsilon_2^T + \varepsilon_3^T - \varepsilon_4^T \right] E
\]

- If we are careful how these extra terms are combined in the bridge equation, it is possible to cancel them out.
- This is called bridge compensation.
Application:

- **Half-bridge compensation:**

  ![Half-bridge diagram]

  \[ e = \frac{GF}{4} (\varepsilon_1 - \varepsilon_2)E + \frac{GF}{4} (\varepsilon_1^T - \varepsilon_2^T)E \]

  since gages #3 and #4 are dummy resistances and therefore experience no strain or temperature changes (if located together).

- If gages experience same temperature change, then the effects are automatically cancelled in the bridge.

- **RULE #1:** equal changes in adjacent arms will cancel out.

- **NOTE:** Single gages cannot be compensated...
Leadwire Effects

- When gages are located long distances from the instrumentation, the effect of the added leadwire resistance can unbalance the bridge:

- For 100 ft of 26 AWG wire in two leadwires:
  \[ R = \frac{40.8 \times 100}{1000} \times 2 = 8.16 \text{ Ohms} \]
  compared to a 120 Ohm gage.

- A 3-wire hookup can eliminate the unbalance problem:

  Since each leadwire resistance appears in adjacent arms (RULE #1)

This looks like a duplicated wire but it really puts a leadwire in AA’E and another in ABC.
Leadwire Desensitization

- Added leadwire resistance in an arm will also desensitize the gage. Consider a single arm:

\[ e = \frac{1}{4} \frac{GF}{1 + \beta} \varepsilon E = \frac{1}{4} GF^* \varepsilon E \]

- We can correct by defining a “new” gage factor:

\[ GF^* = \frac{GF}{1 + \beta} \approx GF \ (1 - \beta) \]

- Example:

\[ R_s = 2 \times 300 \times \frac{64.9}{1000} = 38.9 \text{ Ohms} \]

\[ \beta = \frac{38.9}{120} = 0.324 \]

\[ GF^* = \frac{GF}{1 + \beta} = \frac{2.0}{1.324} = 1.51 \]

Two 300 ft lengths of 28 AWG wire for a 120 Ohm gage with initial GF=2.0

New GF*
Other Issues

- Bridge balancing: how do you get rid of any initial unbalance in the bridge due to unequal initial gage resistances?

- Bridge switching: how can you use a single Wheatstone Bridge instrument in order to read outputs from several different strain gages?
  - Interbridge switching
  - Intrabridge switching
Wheatstone Bridge Applications

- Gages can be arranged in a Wheatstone Bridge to cancel some strain components and to amplify others:

Table 5. Bridge Configurations for Uniaxial Members

<table>
<thead>
<tr>
<th>No.</th>
<th>K</th>
<th>Configuration</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>1</td>
<td></td>
<td>Must use dummy gage in an adjacent arm (2 or 4) to achieve temperature compensation</td>
</tr>
<tr>
<td>A-2</td>
<td>2</td>
<td></td>
<td>Rejects bending strain but not temperature compensated; must add dummy gages in arms 2 &amp; 4 to compensate for temperature.</td>
</tr>
<tr>
<td>A-3</td>
<td>(1+ν)</td>
<td></td>
<td>Temperature compensated but sensitive to bending strains</td>
</tr>
<tr>
<td>A-4</td>
<td>2(1+ν)</td>
<td></td>
<td><strong>Best</strong>: compensates for temperature and rejects bending strain.</td>
</tr>
</tbody>
</table>
### Wheatstone Bridge Applications - cont’d

- **Bending applications:**

  Table 6. Bridge Configurations for Flexural Members

<table>
<thead>
<tr>
<th>No.</th>
<th>K</th>
<th>Configuration</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-1</td>
<td>1</td>
<td><img src="1" alt="Configuration" /></td>
<td>Also responds equally to axial strains; must use dummy gage in an adjacent arm (2 or 4) to achieve temperature compensation</td>
</tr>
<tr>
<td>F-2</td>
<td>2</td>
<td><img src="2" alt="Configuration" /></td>
<td>Half-bridge; rejects axial strain and is temperature compensated; dummy resistors in arms 3 &amp; 4 can be in strain indicator.</td>
</tr>
<tr>
<td>F-3</td>
<td>4</td>
<td><img src="3" alt="Configuration" /></td>
<td><strong>Best:</strong> Max sensitivity to bending; rejects axial strains; temperature compensated.</td>
</tr>
<tr>
<td>F-4</td>
<td>2(1+ν)</td>
<td><img src="4" alt="Configuration" /></td>
<td>Adequate, but not as good as F-3; compensates for temperature and rejects axial strain.</td>
</tr>
</tbody>
</table>
Wheatstone Bridge Applications - cont’d

- Torsion applications:

Table 7. Bridge Configuration for Torsion Members

<table>
<thead>
<tr>
<th>No.</th>
<th>K</th>
<th>Configuration</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1</td>
<td>2</td>
<td><img src="image1" alt="Diagram" /></td>
<td>Half Bridge: Gages at ±45° to centerline sense principal strains which are equal &amp; opposite for pure torsion; bending or axial force induces equal strains and is rejected; arms are temperature compensated.</td>
</tr>
<tr>
<td>T-2</td>
<td>4</td>
<td><img src="image2" alt="Diagram" /></td>
<td><strong>Best</strong>: full-bridge version of T-1; rejects axial and bending strain and is temperature compensated.</td>
</tr>
</tbody>
</table>
Wheatstone Bridge Applications - cont’d

- Rings loaded diametrically are very popular as a means to transduce load by converting it into a proportional strain:

- How would one wire the gages to best advantage in a Wheatstone Bridge?