# The Standard Atmosphere

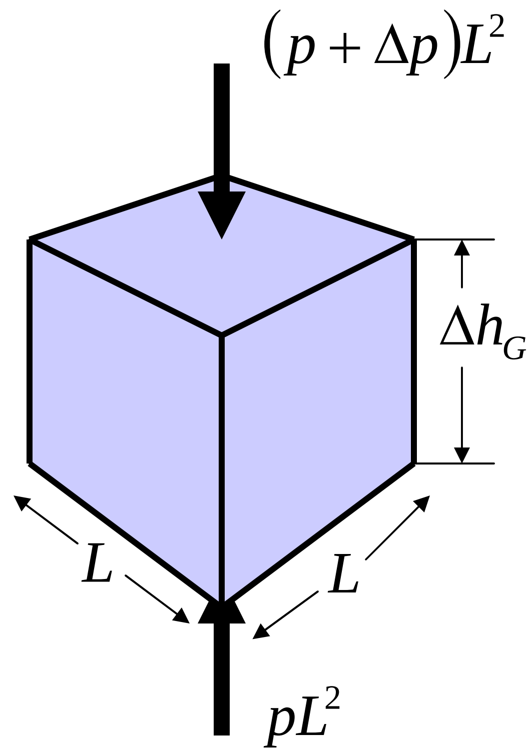
AE 1350

# Weather and the Atmosphere

- Weather varies across location and time
- A standard atmosphere is defined for the Earth
  - We can say an airplane goes "100 mph" – and this will have *meaning*, since this implies we are talking about 100 mph in standard atmospheric conditions
    - Goes faster when air is less dense, has a tail wind, etc.
  - The standard atmosphere model is only a function of altitude (lot latitude/longitude or time) and has no wind
- In terms of quantities we've been using
  - Pressure ( $p$ ) in pounds per square foot, meters per $m^2$
  - Density ( $\rho$ ) in slugs per cubic foot, kilogram per $m^3$
  - Temperature ( $T$ ) degrees Rankine, Fahrenheit, etc.

# Hyrdostatic Equation

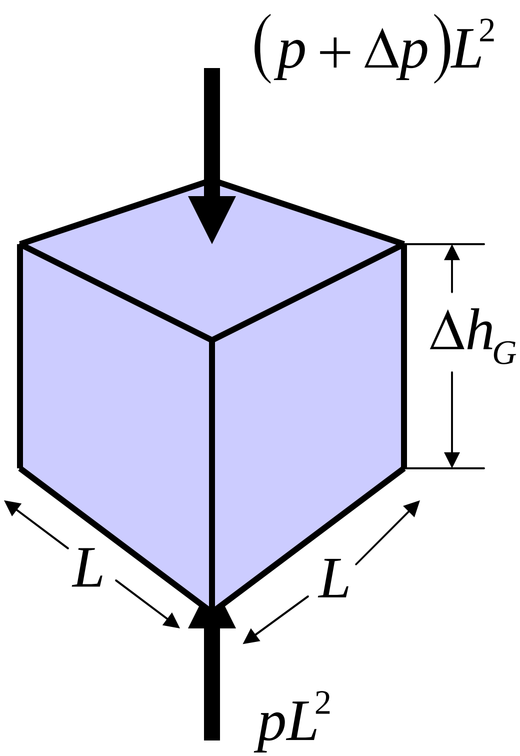
- Standard atmosphere uses a model that accounts for pressure necessary to support the air above
- Consider a cube of air:




$h$ is altitude
$L$ is length/width of cube

# Hyrdostatic Equation

- Force must all balance
  (from above and below plus gravity)




$$pL^2 = (p+\Delta p)L^2 + \rho g \Delta h_G L^2$$

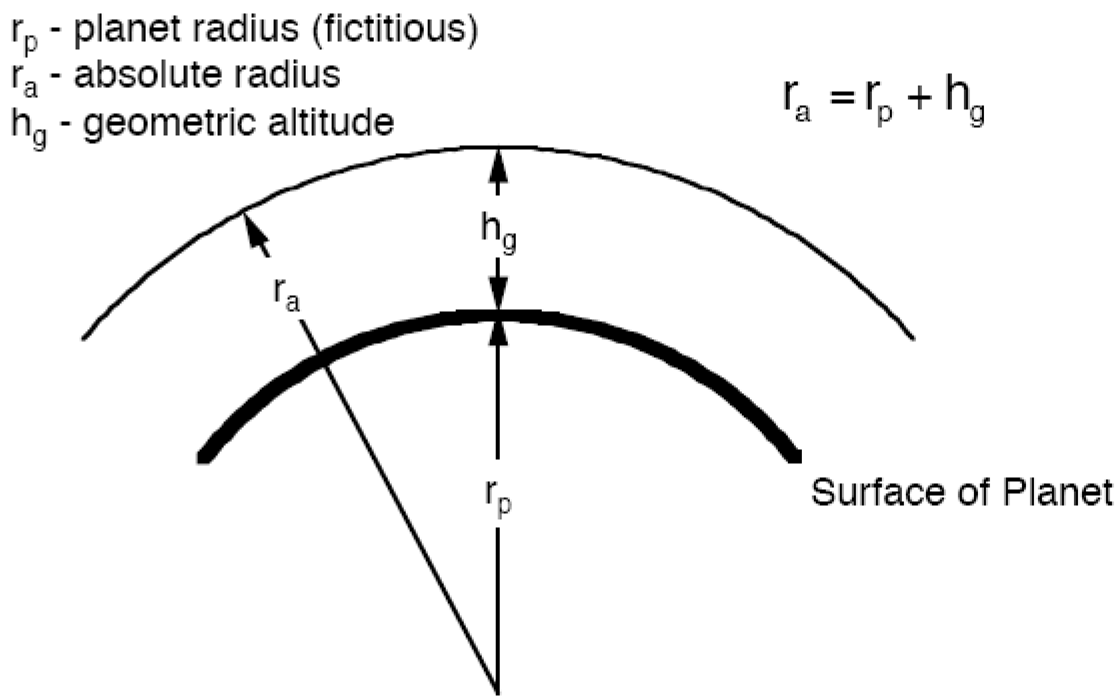
$g$ is acceleration due to gravity:

$$g = g_0 \left( \frac{r}{r+h_G} \right)^2$$

Gives us the hydrostatic equation:
go up and the pressure drops

$$\Delta p = -\rho g \Delta h_G$$

# Geometric Altitude ($h_g$)




- Geometric altitude is the geometric height above a sea-level reference
- The gravitational attraction (which varies inversely with the square of absolute radius) at this reference is defined as $g_0$

# Geopotential Altitude

- The inverse squared gravity law makes the differential equations a mess
- Fix by defining a new type of altitude:  so gravity is effectively constant in the hydrostatic equation

$$\Delta p = -\rho g \Delta h_G \quad \text{becomes} \quad \Delta p = -\rho g_0 \Delta h$$

- To make this happen:

$$g \Delta h_G = g_0 \Delta h$$

$$\Delta h = \frac{g}{g_0} \Delta h_G$$

$$\Delta h = \frac{r^2}{\left(r + h_G\right)^2} \Delta h_G$$

So…

Both measures of altitude are nearly the same at low altitudes

Be careful which is being used!

# Need to Relate Density and Pressure

- Model assumes air behaves like a perfect gas (intermolecular forces negligible)
- Equation of state:

$$\boxed{p = \rho R T}$$

where $R$ is specific gas constant

$$R = 287 \frac{J}{kg \cdot {}^{o}K}$$

$$R = 1716 \frac{ft \cdot lb}{slug \cdot {}^{o}R}$$

# Put it all together…

$$\Delta p = -\rho g_0 \Delta h$$

$$\frac{\Delta p}{p} = \frac{-\rho g_0 \Delta h}{p}$$

$$\frac{dp}{p} = \frac{-\rho g_0 dh}{\rho R T}$$

$$\boxed{\frac{dp}{p} = -\frac{g_0}{RT} dh}$$

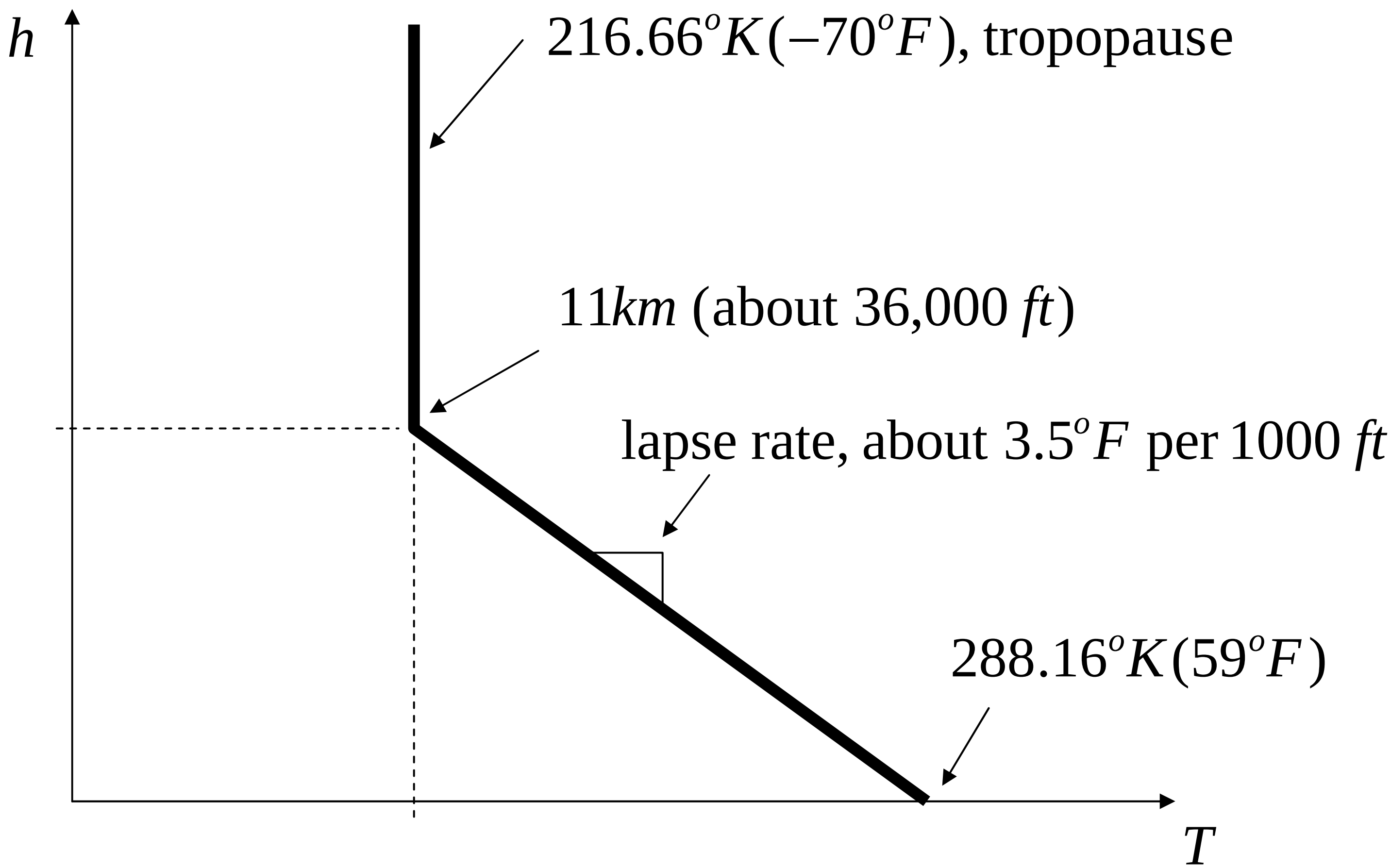
Now we have pressure as a function of temperature and altitude

# The Standard Atmosphere Model

- The standard atmosphere is a temperature profile (function of altitude)
- Use $\frac{dp}{p} = -\frac{g_0}{RT} dh$ → pressure
- Use $\rho = \frac{p}{RT}$ → density
- Convert from geopotential altitude to regular altitude if needed

# Temperature profile (first 20 km)

1976 U.S. Standard Atmosphere



# In the Tropopause…

$T$ is a constant

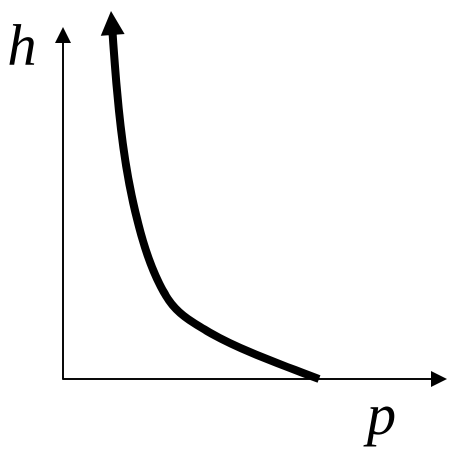
$$\frac{dp}{p} = -\frac{g_0}{RT} dh$$

$$\int_{p_1}^{p} \frac{dp}{p} = -\frac{g_0}{RT} \int_{h_1}^{h} dh$$

$$\ln \frac{p}{p_1} = -\frac{g_0}{RT} (h - h_1)$$

$$\frac{p}{p_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$

$$\frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$



# Below the Tropopause…

$T$ is proportional to $h$, by lapse rate ($a$)

$$a = \frac{dT}{dh}$$

$$dh = \frac{dT}{a}$$

$$\frac{dp}{p} = -\frac{g_0}{RT}dh$$

$$\frac{dp}{p} = -\frac{g_0}{aR}\frac{dT}{T}$$

$$\int_{p_1}^{p}\frac{dp}{p} = -\frac{g_0}{aR}\int_{T_1}^{T}\frac{dT}{T}$$

$$\ln\frac{p}{p_1} = -\frac{g_0}{aR}\ln\frac{T}{T_1}$$

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}}$$

$$\frac{p}{p_1} = \frac{\rho RT}{\rho_1 RT_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)}$$

$$T = T_1 + a\left(h - h_1\right)$$

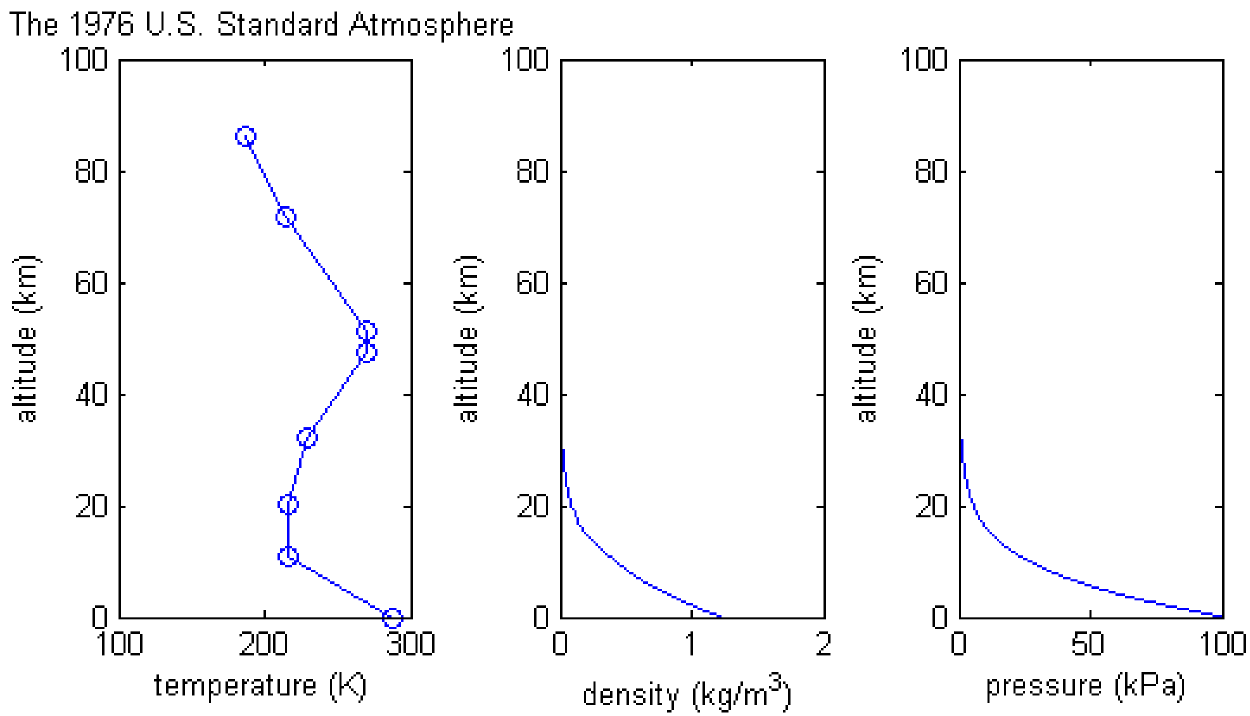
# Other Constants Needed

1976 U.S. Standard Atmosphere

- To start the integration, need values at $h = 0$

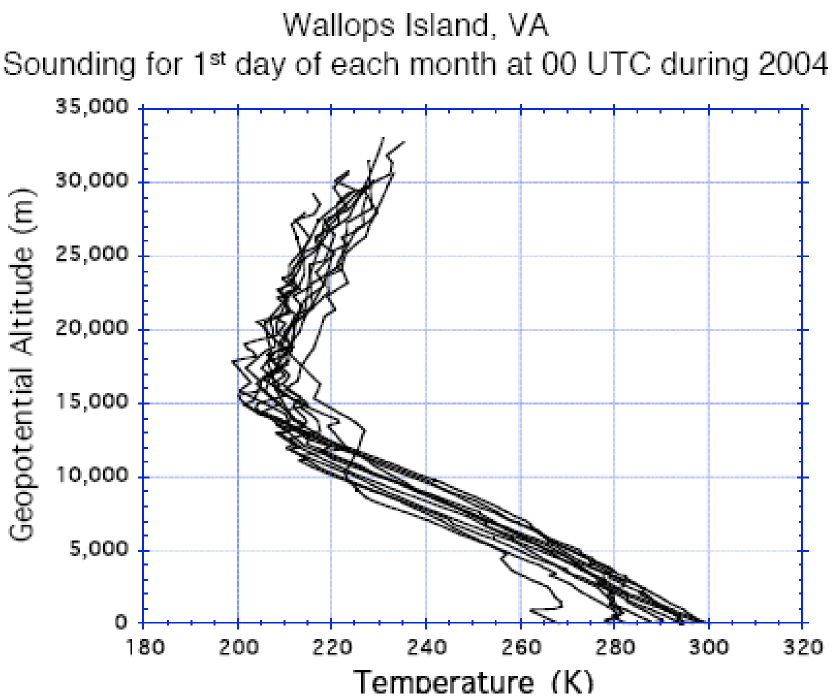
$$p(0) = 101325 \frac{N}{m^2} = 2112.2 \frac{lb}{ft^2}$$

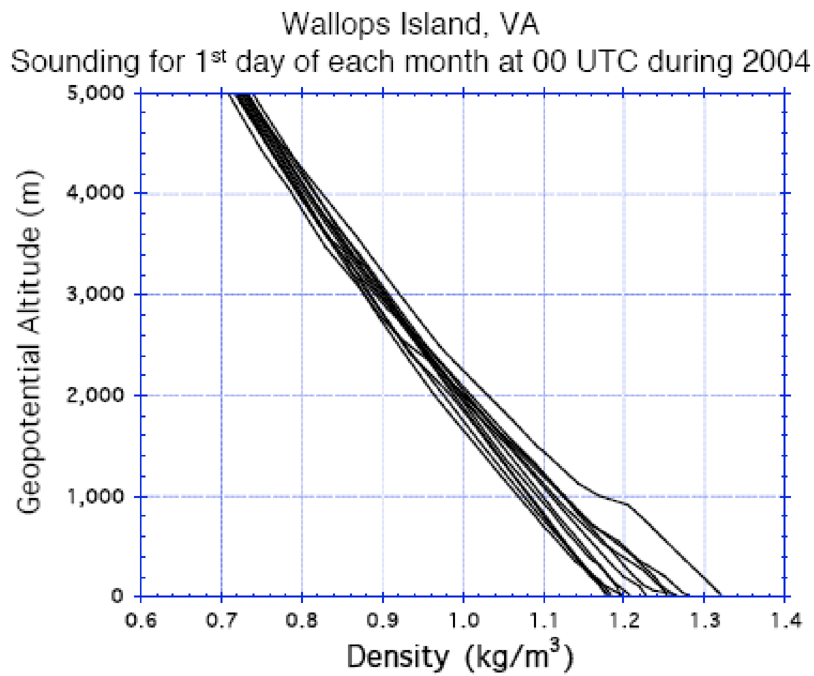
$$\rho(0) = 1.225 \frac{kg}{m^3} = 0.002377 \frac{slug}{ft^3}$$



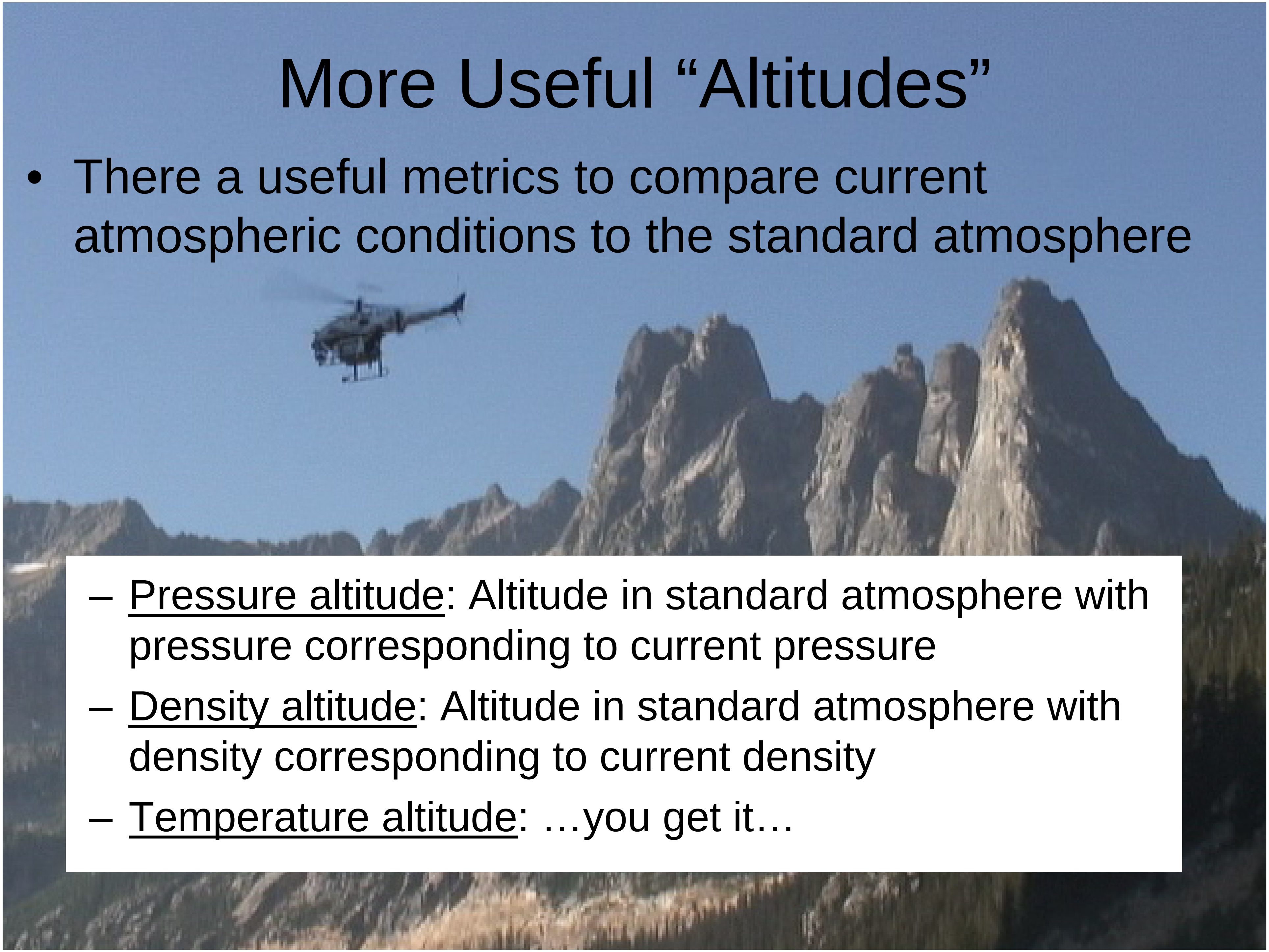

from Wikipedia

# Flight Measured Atmospheric Properties



# More Useful “Altitudes”



- There a useful metrics to compare current atmospheric conditions to the standard atmosphere
  - Pressure altitude: Altitude in standard atmosphere with pressure corresponding to current pressure
  - Density altitude: Altitude in standard atmosphere with density corresponding to current density
  - Temperature altitude: …you get it…

# 1976 U.S. Standard Atmosphere is in your text, Appendix A & B

| Geopotential Altitude, ft | Altitude, ft | Temp, $^{o}$R | Pressure, lb/ft$^2$ | Density, slug/ft$^3$ |
|---|---|---|---|---|
| 0 | 0 | 518.69 | 2116.2 | 0.0023769 |
| 1000 | 1000 | 515.12 | 2040.9 | 0.0022743 |
| 2000 | 2000 | 511.56 | 1967.7 | 0.0022409 |