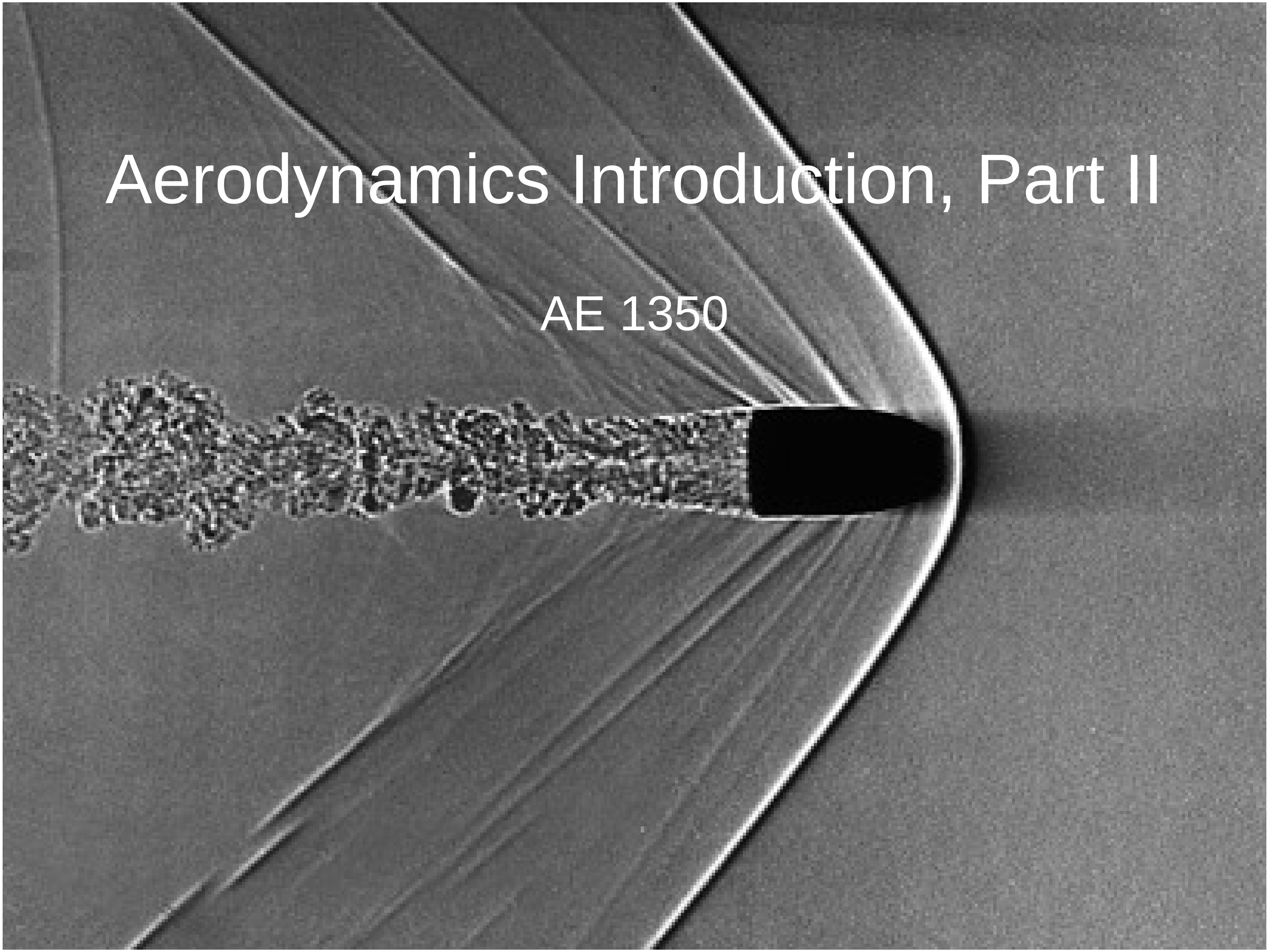


Aerodynamics Introduction, Part II

AE 1350

So far…

- Continuity and conservation of Momentum gave us:

$$AV = Const$$

$$\frac{1}{2}\rho V^2 + p = Const$$

- For compressible flow, we need another equation to allow us to solve for what happens to density…

$$\rho AV = Const$$

$$\rho V dV + dp = 0$$

Compressible Flow

- As stated earlier, when the velocity exceeds about $M = 0.4$, the flow can no longer be assumed to be incompressible
- In high speed flows, significant changes in temperature are associated with significant changes in energy (flow speed)
- In addition to the previous relations, we will need to make use of the first law of thermodynamics

Some Definitions

- Specific internal energy:
  Energy stored in random motion of molecules, per unit mass
  - For diatomic molecules (two atoms), $e = 5/2\ RT$ (function of Temperature!)
    we'll use $e = C_v T$, where $C_v = 5/2\ R$ for air
- Specific enthalpy
  Energy + pressure*volume, per unit mass:
  $h = e + p/\rho = e + RT = 7/2RT = C_p T$, $C_p = 7/2\ R$
- Ratio of specific heats:
  $\gamma = C_p/C_v = 7/5 = 1.4$ for air

First Law of Thermodynamics

- Change in the specific internal energy of a system is due to heat added to the system, and work done on the system

$$de = \delta q + \delta w$$

Heat added per
Unit mass

Work done on the
System per unit mass
due to body forces
Such as gravity, and
Pressure forces

First Law of Thermodynamics

- Neglecting forces such as gravity, electric, and magnetic…
- The work done per unit mass is:

$$\delta w = -p\, dV\ /\ \mathrm{Mass} = -p\, d\left(\frac{1}{\rho}\right)$$

- The first law becomes:

Internal energy: $$de = \delta q - pd\left(\frac{1}{\rho}\right)$$

Enthalpy: $$dh = d(e + \frac{p}{\rho}) = de + pd\left(\frac{1}{\rho}\right) + \frac{1}{\rho}dp = \delta q + \frac{1}{\rho}dp$$

Thermodynamic Process

- The means by which changes in the fluid properties of a system take place
  - Constant volume process: gas inside a rigid volume (e.g., propellant tank). As heat is added, P and T will change. By definition, $dv = 0$

$$\text{We define } C_v = \frac{dq}{dT} \text{ at constant volume}$$

$$\text{First law: } de = \delta q = C_v dT$$

Assuming that C_v is not a function of temperature and letting $e = 0$ at $T = 0$ yields,

$$\boxed{e = C_v T}$$

  - Constant pressure process: piston system in which pressure is maintained as heat is added. As heat is added to this system, T and ρ will change. By definition, $dp = 0$

$$\text{We define } C_P = \frac{dq}{dT} \text{ at constant pressure}$$

$$\text{First law: } dh = \delta q = C_P dT$$

Assuming that C_P is not a function of temperature and letting $h = 0$ at $T = 0$ yields,

$$\boxed{h = C_P T}$$

These equations are derived from the First Law of Thermodynamics into which the definitions of specific heat have been applied. While not specifically proven here, they are applicable to any process in which a perfect gas is used.

Adiabatic Process

- An adiabatic process is one in which there is no heat addition or removal (work is done by the pressure forces, but no heat is added)
- Examples of adiabatic flow are: Flow over a wing (outside the boundary layer), flow through a propeller or a turbine
- Example of a non-adiabatic flow: Flow through a combustor or furnace, flow within the boundary layer where the wall is cooler or hotter than the fluid
- First law becomes just: $$dh = \frac{1}{\rho} dp$$

First Law for Adiabatic Inviscid Flows

- Recall Euler's equation for conservation of momentum in a stream tube:

$$\rho V dV + dp = 0$$

$$V dV + \frac{1}{\rho} dp = 0$$

$$dh = \frac{1}{\rho} dp$$

1st law!

$$V dV + dh = 0$$

$$\int V dV + \int dh = Const$$

$$\frac{V^2}{2} + h = Const = h_0$$

"stagnation enthalpy"

First Law for Adiabatic Inviscid Flows

$$h + \frac{V^2}{2} = h_0$$ "stagnation enthalpy"

Or,

$$C_p T + \frac{V^2}{2} = C_p T_0$$ where T_0 is called stagnation temperature

Use $h = e + p/\rho = C_v T + \frac{p}{\rho}$

$$C_v T + \frac{p}{\rho} + \frac{V^2}{2} = C_p T_0$$

→ Generalization of Bernoulli's Equation for compressible flows

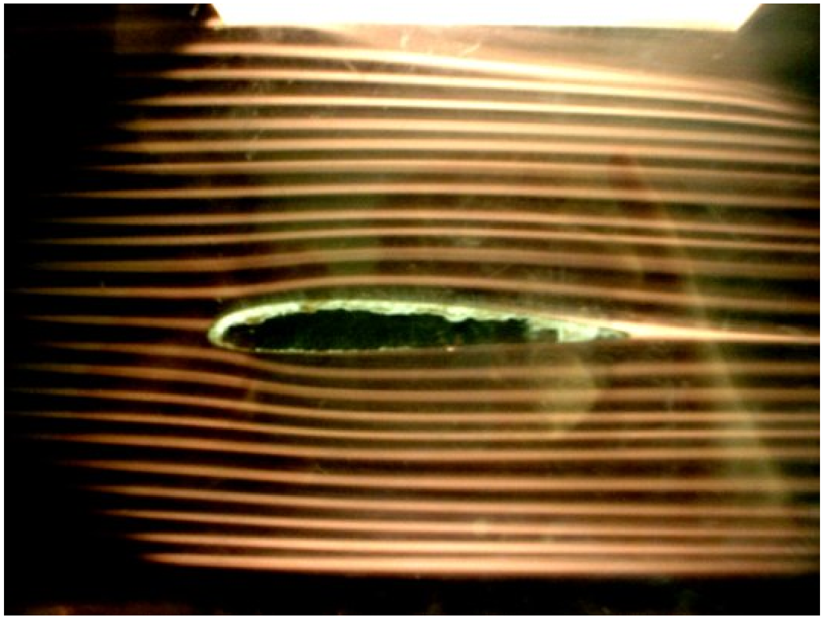
Internal Energy — "Pressure work" — Kinetic Energy

Reversible Flow

- A reversible flow is one in which the <u>system</u> (i.e. collection of fluid particles of interest) and the <u>environment</u> (i.e. surrounding particles), can <u>both</u> be restored to their original condition
  - no friction or dissipative effects
- Example of a reversible process:
  - Slow compression of air in a balloon does work on the air inside the balloon, and takes away energy from the surroundings - When the balloon is allowed to expand, the air inside and the surrounding air are both restored to original conditions
- Example of an irreversible process:
  - Heat flows from hot to cold, never in the opposite direction; Most conductive and viscous processes are irreversible
  - Flow through a boundary layer, viscosity creates skin friction

Isentropic Flow

- Adiabatic: no heat is added or taken away ($\delta q=0$)
- Reversible: no frictional or other dissipative effects
- Isentropic: BOTH! (Adiabatic + Reversible)



Isentropic Flow

Internal energy:

$$de = C_v dT = \cancel{\delta q} - pd\left(\frac{1}{\rho}\right)$$

$$-pd\left(\frac{1}{\rho}\right) = C_v dT$$

Enthalpy:

$$dh = C_p dT = \cancel{\delta q} + \frac{1}{\rho} dp$$

$$\frac{1}{\rho} dp = C_p dT$$

$$\frac{-pd\left(\frac{1}{\rho}\right)}{\frac{1}{\rho} dp} = \frac{C_v dT}{C_p dT} = \frac{1}{\gamma}$$

Isentropic Flow

$$\frac{-pd\left(\frac{1}{\rho}\right)}{\frac{1}{\rho}dp} = \frac{1}{\gamma} \quad \longrightarrow \quad \frac{dp}{p} = -\gamma \frac{d\left(\frac{1}{\rho}\right)}{\frac{1}{\rho}}$$

$$\int_1^2 \frac{dp}{p} = -\gamma \int_1^2 \frac{d\left(\frac{1}{\rho}\right)}{\frac{1}{\rho}}$$

$$\ln\left(\frac{p_2}{p_1}\right) = -\gamma \ln\left(\frac{\frac{1}{\rho_2}}{\frac{1}{\rho_1}}\right)$$

$$\frac{p_2}{p_1} = \left(\frac{\frac{1}{\rho_2}}{\frac{1}{\rho_1}}\right)^{-\gamma}$$

A relationship between density and pressure we can use for compressible flow!
(as long as it's isentropic…)

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

Isentropic Flow

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

From ideal gas law: $p = \rho RT$

$$\frac{p_2}{p_1} = \left(\frac{p_2}{p_1}\frac{RT_1}{RT_2}\right)^{\gamma}$$

$$\left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{-\gamma}$$

Can find temperature also if we need it!

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

Isentropic Flow Summary

$$h + \frac{V^2}{2} = C_p T + \frac{V^2}{2} = C_p T_0 = \text{Constant}$$

$$\frac{p}{\rho^\gamma} = \text{Constant}$$

$$\frac{\rho}{T^{\frac{1}{\gamma-1}}} = \text{Constant}$$

More on the Speed of Sound

- From thermodynamics, and compressible flow theory, sound travels at the following speed:

$$a = \sqrt{\gamma R T}$$

- where,
  - a is Speed of Sound
  - γ is Ratio of Specific Heats (1.4 for air)
  - R is Gas Constant
  - T is Temperature (in ^{o}K or degrees ^{o}R)

- So, really only depends on Temperature!

As a Function of Mach Number…

- These same relations can be expressed as a function of Mach number as:

$$C_p T + \frac{V^2}{2} = C_p T_0$$

$$C_p = \gamma C_V = \frac{\gamma R}{\gamma - 1}$$

$$\frac{\gamma R T}{\gamma - 1} + \frac{V^2}{2} = \frac{\gamma R T_0}{\gamma - 1}$$

$$1 + \frac{\gamma - 1}{2}\frac{V^2}{\gamma R T} = \frac{T_0}{T}$$

a^2

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{\gamma-1}}$$

$$\frac{P_0}{P} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

The quantities P_0, ρ_0, and T_0 are called stagnation pressure, stagnation density and stagnation temperature. They represent the properties the flow would have if brought to rest reversibly and adiabatically.

Note that:

$$\frac{P_0}{P} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

Using Mach Number in Energy Equations

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

"Mach meter" (M<1)
based on pressure measurements
Interestingly, doesn't require temp

Requires static and total pressure, like a Pitot tube
(although Pitot tube only needed the difference in pressures)

Summary

- For steady incompressible flow (ρ= constant) of a frictionless (inviscid) fluid, we need only concern ourselves with conservation of mass (Continuity) and momentum (Bernoulli):

$$\mathrm{AV} = \text{constant}$$
$$\frac{1}{2}\rho \mathrm{V}^2 + \mathrm{P} = \text{constant}$$

- For steady, adiabatic and frictionless (isentropic) compressible flow, P, T, ρ and V are all variables, and are related through conservation of mass (Continuity), momentum (Energy), the first law of thermodynamics (Isentropic flow relations) and the Equation of State

$$\rho \mathrm{AV} = \text{constant}$$
$$\mathrm{h} + \frac{\mathrm{V}^2}{2} = \text{constant}$$
$$\frac{\mathrm{P}_0}{\mathrm{P}} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left(\frac{\mathrm{T}_0}{\mathrm{T}}\right)^{\frac{\gamma}{\gamma-1}}$$
$$\mathrm{P} = \rho \mathrm{RT}$$