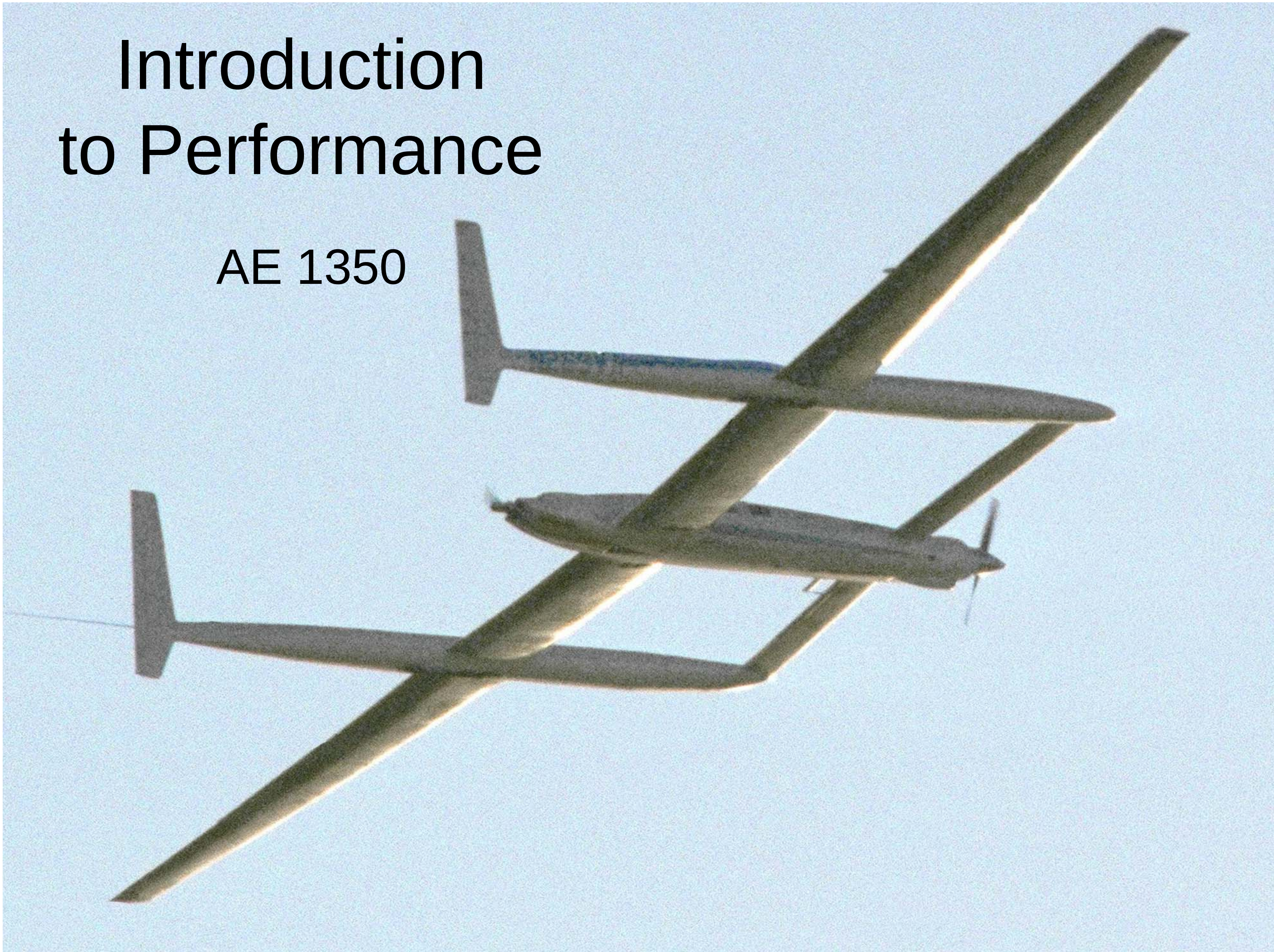
Introduction to Performance

AE 1350



Performance: A measure of how well a device does its job

Airplane Performance Examples

Speed -> how fast/slow can it go?

Rate of Climb -> how fast can it go up?

Ceiling -> how high can it go?

Range -> how far can it go?

Endurance -> for how long can it fly?

Takeoff/Landing -> how much runway does it need?

Turning -> what is the minimum turn radius?

Helicopter Performance Examples

Hover Capability -> how much weight can it lift vertically?

Speed -> how fast can it go?

Rate of Climb -> how fast can it go up?

Ceiling -> how high can it go?

Range -> how far can it go?

Endurance -> for how long can it fly?

Launch Vehicle/Rocket Performance Examples

Payload Mass to Low-Earth Orbit

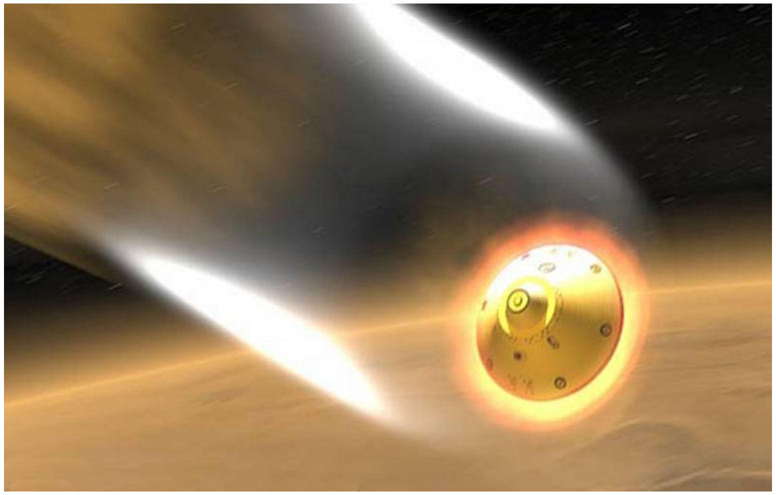
Payload Mass to Geosynchronous Orbit

Change in Velocity ΔV

Entry, Descent, and Landing Vehicle Performance Examples

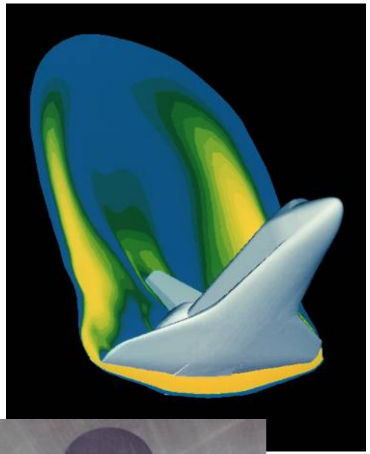
Landed Mass

Landing Accuracy



How is Aerospace Vehicle Performance Calculated?

Aerospace vehicle performance is determined by using one or more of the following:



Mathematical Modeling

- Computational Fluid Dynamics
- Classical aero/propulsive/mass analyses

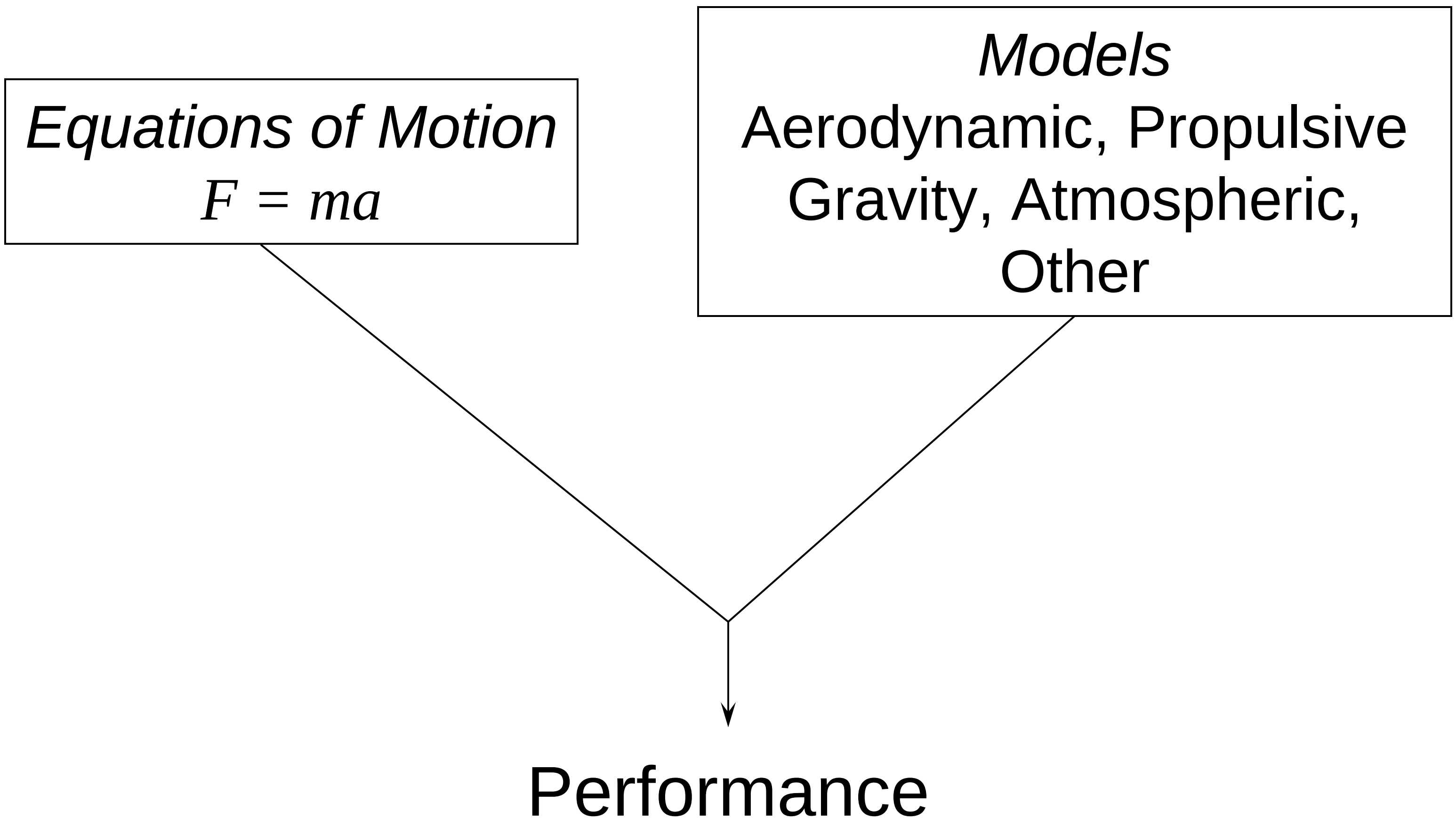
Ground Testing

- Wind tunnel testing
- Static engine testing

Flight Testing

Airplane Performance

<u>General Approach</u>



Airplane Performance

Aerodynamic Models

The lift, L, and drag, D, of the airplane can be calculated from:

$$L = \frac{1}{2}\rho V^2 S C_L \qquad D = \frac{1}{2}\rho V^2 S C_D$$

Where:

ρ is the atmospheric density
V is the airspeed
S is the wing area
C_L is the lift coefficient
C_D is the drag coefficient

Airplane Performance

Aerodynamic Models - continued…

We will model the relationship between the lift and drag of an airplane through the drag polar:

$$C_D = C_{D0} + KC_L^2$$

Where we know the values of C_{D0} and K for our airplane.

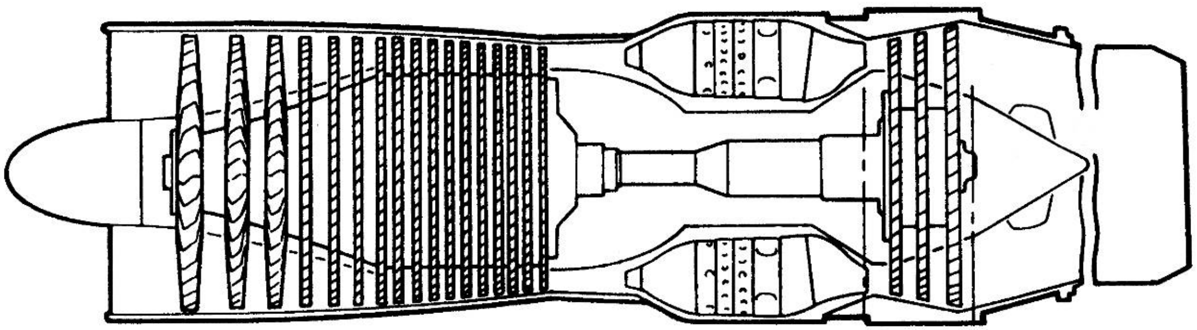
The lift coefficient, C_L, has a known maximum value:

$$C_{L,Max}$$

Airplane Performance

Propulsion Models

In this introduction to performance lecture we will assume all our airplanes are using turbojet engines (no bypass):



Graphics source: Anderson, Aircraft Performance and Design

Airplane Performance

Propulsion Models - continued…

For a turbojet engine, maximum thrust, T_{Max}, does not change very much with airspeed, V, while flying at subsonic speeds

For a turbojet engine the maximum thrust, T_{Max}, changes with density as given by:

$$T_{Max} = T_{Max0}\left(\frac{\rho}{\rho_0}\right)$$

Where,

ρ_0 is the atmospheric density at sea level
T_{Max0} is the maximum thrust at sea level

Airplane Performance

Propulsion Models - continued…

How do we quantify the fuel consumption of a turbojet engine? We usually do so by calculating the thrust specific fuel consumption, c_t:

$$c_t = \text{thrust specific fuel consumption}$$

$$= \frac{(\text{weight of fuel consumed for a given time increment})}{(\text{thrust output}) \bullet (\text{time increment})}$$

$$= \frac{\dot{w}_{fuel}}{T}$$

In this lecture we will treat c_t as a constant

At subsonic speeds c_t is constant;
It does not vary much with velocity or altitude

Steady Level Flight

By definition, lift is perpendicular to the wing and V

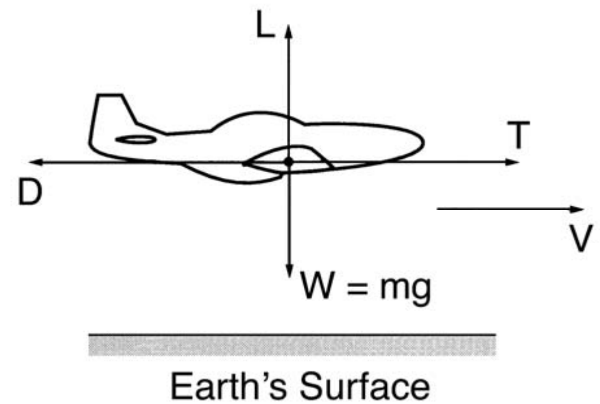
By definition drag is parallel to V

No acceleration (steady), V = constant

Wings level

Horizontal flight (no climb/descent)

Usually the magnitude of L and W are greater than D and T



Summing forces parallel to the flight direction:

$T - D = 0$ or equivalently $T = D$

Summing forces perpendicular to the flight direction:

$L - W = 0$ or equivalently $L = W$

Graphics source: Anderson, Aircraft Performance and Design

Stall Speed

Let us determine how slow we can fly in steady level flight.
In other words, we want to determine the stall speed, V_{stall}

$$L = \frac{1}{2}\rho V^2 S C_L = W$$

$$V = \sqrt{\frac{2W}{\rho S C_L}}$$

How do we minimize V for a given airplane?
By flying at $C_{L,Max}$!

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,Max}}}$$

Stall Speed

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,Max}}}$$

How does the various parameters in this equation:

$$W, \rho, S, C_{L,Max}$$

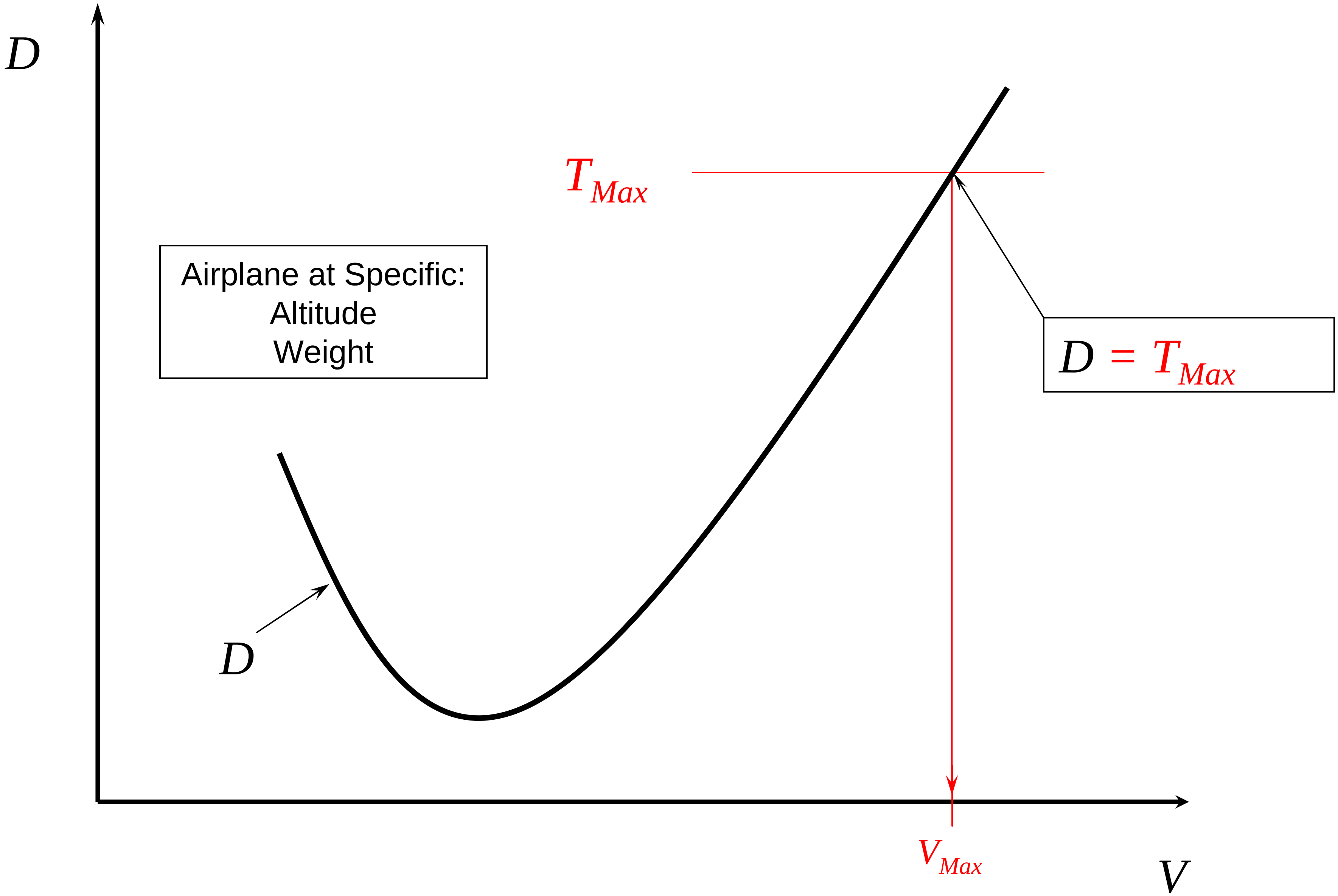
affect V_{stall}?

Numerical Example: Motorglider

m = 300 kg
$W = mg$ = 2,943 N
ρ = 1.225 kg/m^3 (i.e., sea level)
S = 12.5 m^2
$C_{L,Max}$ = 1.5

V_{stall} = 16 m/s

Maximum Speed



Maximum Speed

To determine the maximum speed of a turbojet airplane we start with the equation of motion:

$$T = D$$

But we agreed to model the thrust as:

$$T = T_{Max} = T_{Max0}\left(\frac{\rho}{\rho_0}\right)$$

and determine drag from:

$$D = \frac{1}{2}\rho V^2 S C_D$$

Substituting these equations into the first one gives us:

$$T_{Max0}\left(\frac{\rho}{\rho_0}\right) = \frac{1}{2}\rho V^2 S C_D$$

Maximum Speed

We also agreed to a drag polar model for C_D of the form:

$$C_D = C_{D0} + KC_L^2$$

This equation has C_L in it… Let's see if we can get rid of it by writing C_L in terms of V. From our other equation of motion we already know that:

$$L = \frac{1}{2}\rho V^2 S C_L = W$$

Solving this equation for C_L gives us:

$$C_L = \frac{2W}{\rho V^2 S}$$

Substituting this equation for C_L into our polar model yields:

Maximum Speed

$$C_D = C_{D0} + K \frac{4W^2}{\left(\rho V^2 S\right)^2}$$

Which we can then plug in onto our equation relating thrust and drag:

$$T_{Max0}\left(\frac{\rho}{\rho_0}\right) = \frac{1}{2}\rho V^2 S C_D$$

to get, after some rearranging:

$$T_{Max0}\left(\frac{\rho}{\rho_0}\right) = \frac{1}{2}\rho V^2 S C_{D.0} + \frac{2KW^2}{\rho V^2 S}$$

Multiplying both sides of this equation by V^2 gives us:

Maximum Speed

Which, after some rearranging gives us:

$$\left[\frac{1}{2}\rho S C_{D0}\right]V^4 - \left[T_{Max0}\left(\frac{\rho}{\rho_0}\right)\right]V^2 + \frac{2KW^2}{\rho S} = 0$$

Notice that this is simply a quadratic equation in V^2 of the form:

$$a\left(V^2\right)^2 + bV^2 + c = 0$$

Where,

$$a = \left[\frac{1}{2}\rho S C_{D0}\right] \qquad b = \left[-T_{Max0}\left(\frac{\rho}{\rho_0}\right)\right] \qquad c = \left[\frac{2KW^2}{\rho S}\right]$$

Maximum Speed

This quadratic equation has a solution of the form:

$$V^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

I leave it up to you to complete the gory algebra to show that:

$$V = \sqrt{\frac{T_{Max0}\left(\frac{\rho}{\rho_0}\right) \pm \sqrt{T_{Max0}^2\left(\frac{\rho}{\rho_0}\right)^2 - 4C_{D0}KW^2}}{\rho S C_{D0}}}$$

Finally, we are interested in the maximum speed, V_{Max}, so we pick the positive root in the above equation to yield…

Maximum Speed

$$V_{Max} = \sqrt{\frac{T_{Max0}\left(\frac{\rho}{\rho_0}\right) + \sqrt{T_{Max0}^2\left(\frac{\rho}{\rho_0}\right)^2 - 4C_{D0}KW^2}}{\rho S C_{D0}}}$$

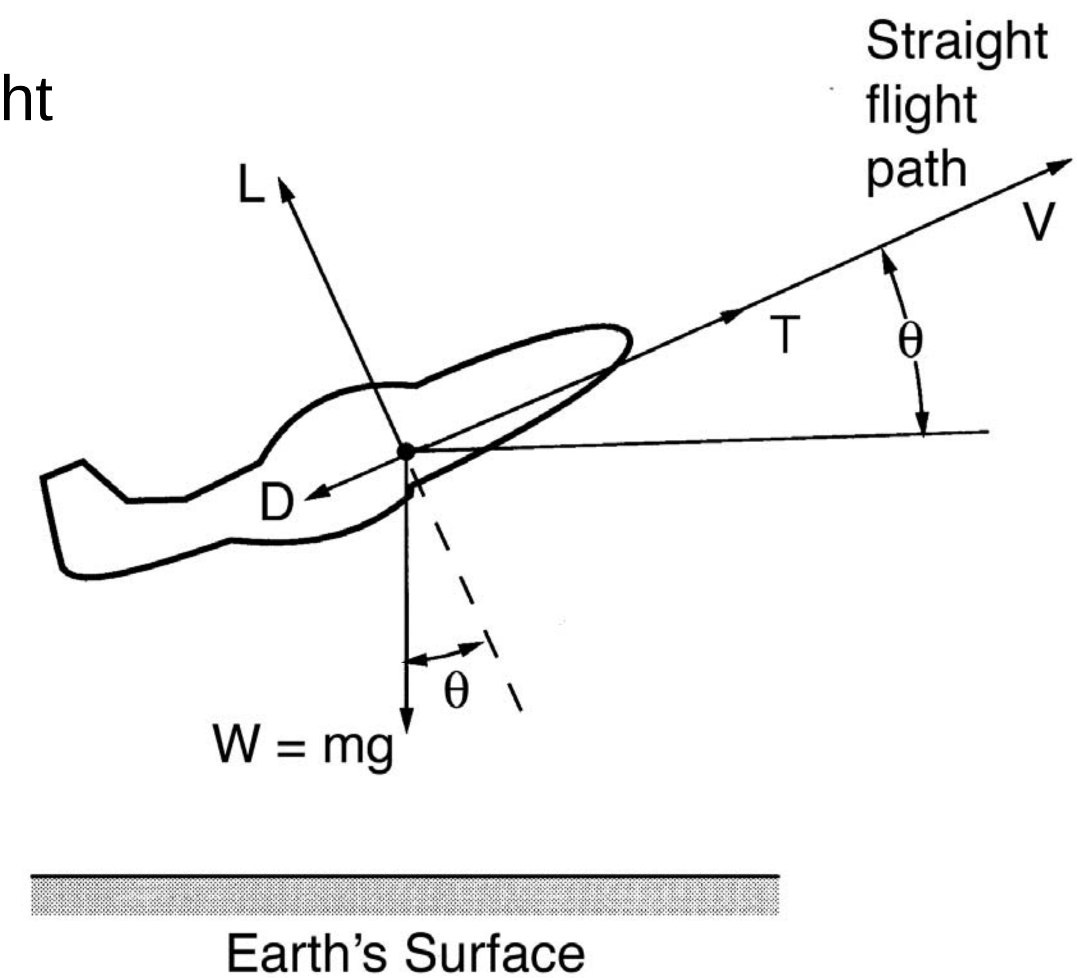
Numerical Example: Motorglider Powered by a Turbojet (Wow!)

m = 300 kg
$W = mg$ = 2,943 N
ρ = 0.6601 kg/m^3 (i.e., 6,000 m altitude)
ρ_0 = 1.225 kg/m^3 (i.e., sea level)
S = 12.5 m^2
C_{D0} = 0.015
K = 0.020
T_{Max0} = 500 N

V_{Max} = 64.7 m/s

Rate of Climb

Steady Climbing Flight



θ - flight path angle with respect to horizon (positive as shown)

Note: arrows not to scale!

Graphics source: Anderson, Aircraft Performance and Design

Rate of Climb

No acceleration (steady)

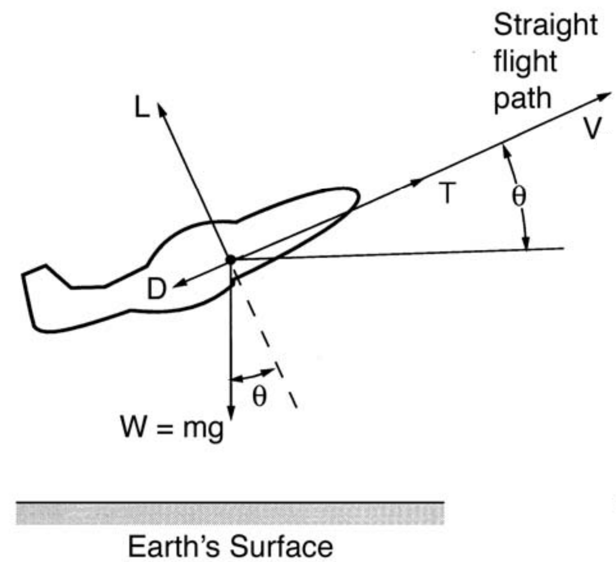
Wings level

Climbing flight at an angle θ to Earth's surface

Thrust parallel to velocity

Summing forces parallel to the flight direction:

$$T - D - W \sin\theta = 0$$



Summing forces perpendicular to the flight direction:

$$L - W \cos\theta = 0$$

Rate of Climb = ROC = $V \sin\theta$

Note that the same equations apply to steady descending flight!

Graphics source: Anderson, Aircraft Performance and Design

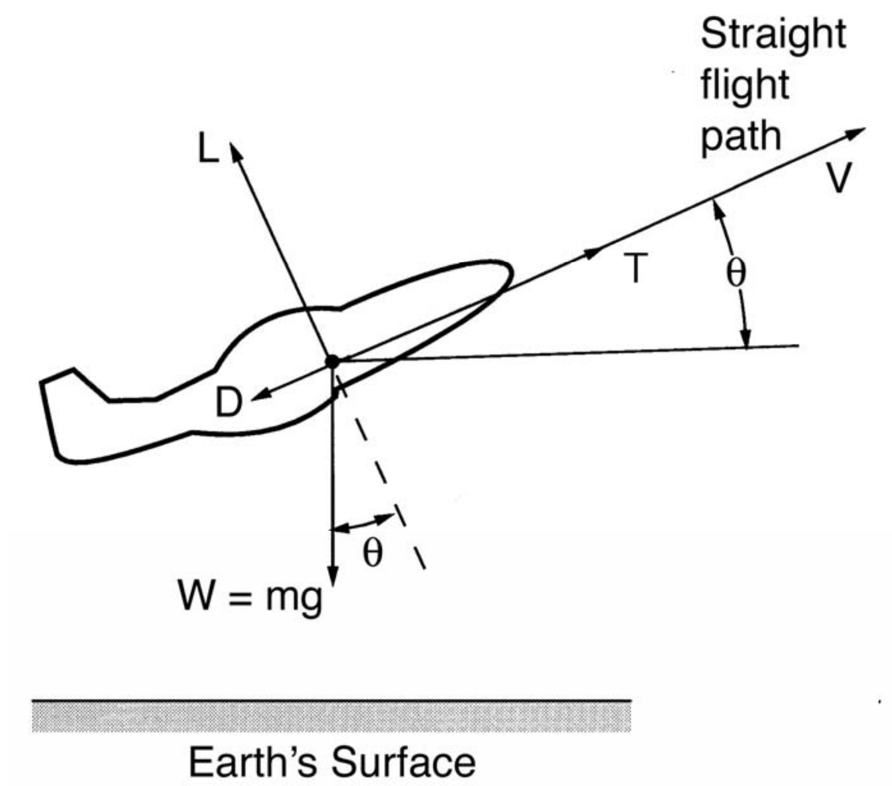
Rate of Climb

Starting with the sum of forces parallel to the flight direction:

$$T - D - W \sin\theta = 0$$

Multiply by V and re-arrange:

$$\frac{TV - DV}{W} = V\sin\theta = ROC$$



Note that:

$$T\,V = P_A = \text{Power Available}$$

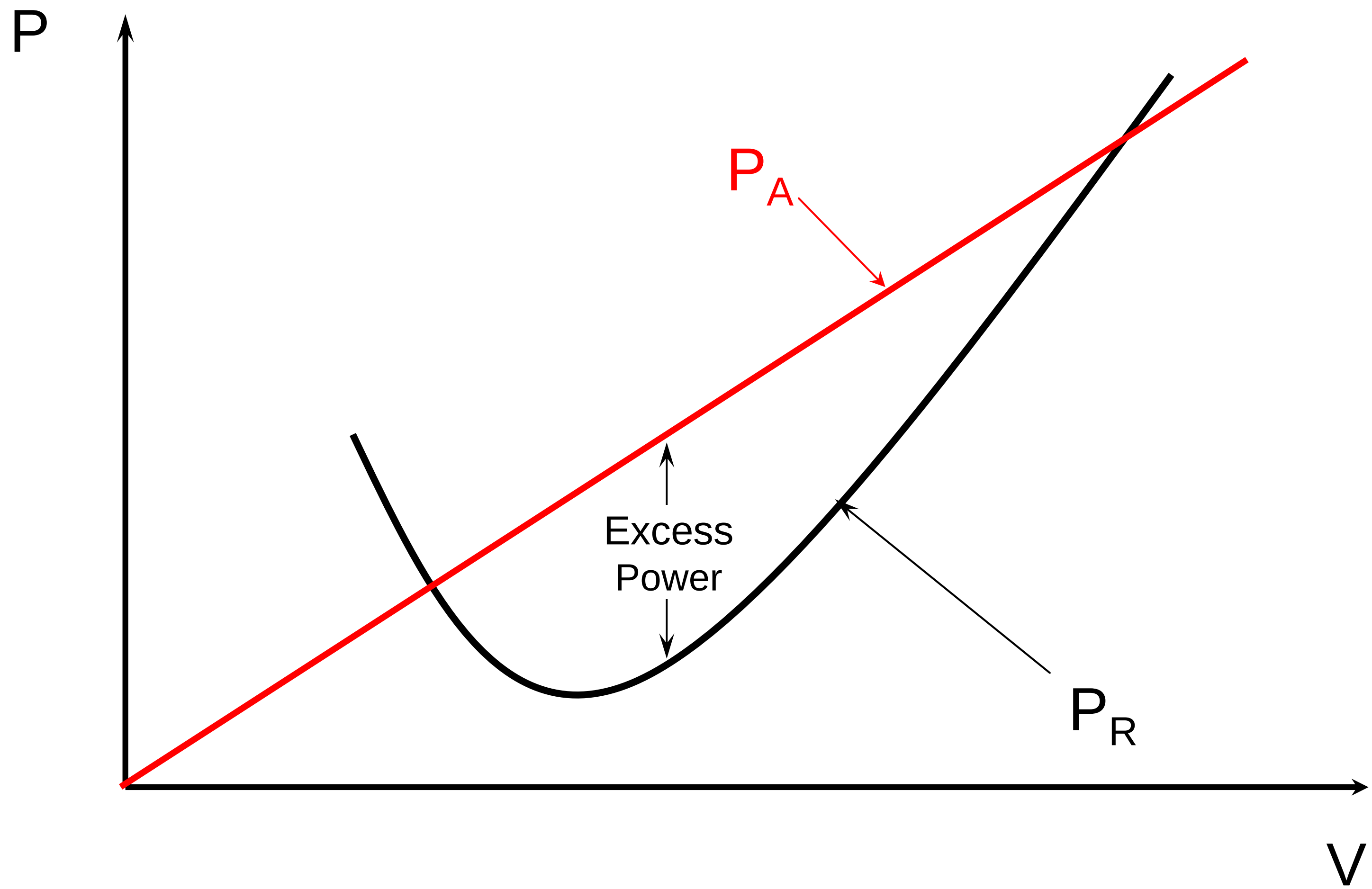
$$D\,V = P_R = \text{Power Required}$$

Thus,

$$ROC = \frac{\text{Power Available} - \text{Power Required}}{W} = \frac{\text{Excess Power}}{W}$$

Graphics source: Anderson, Aircraft Performance and Design

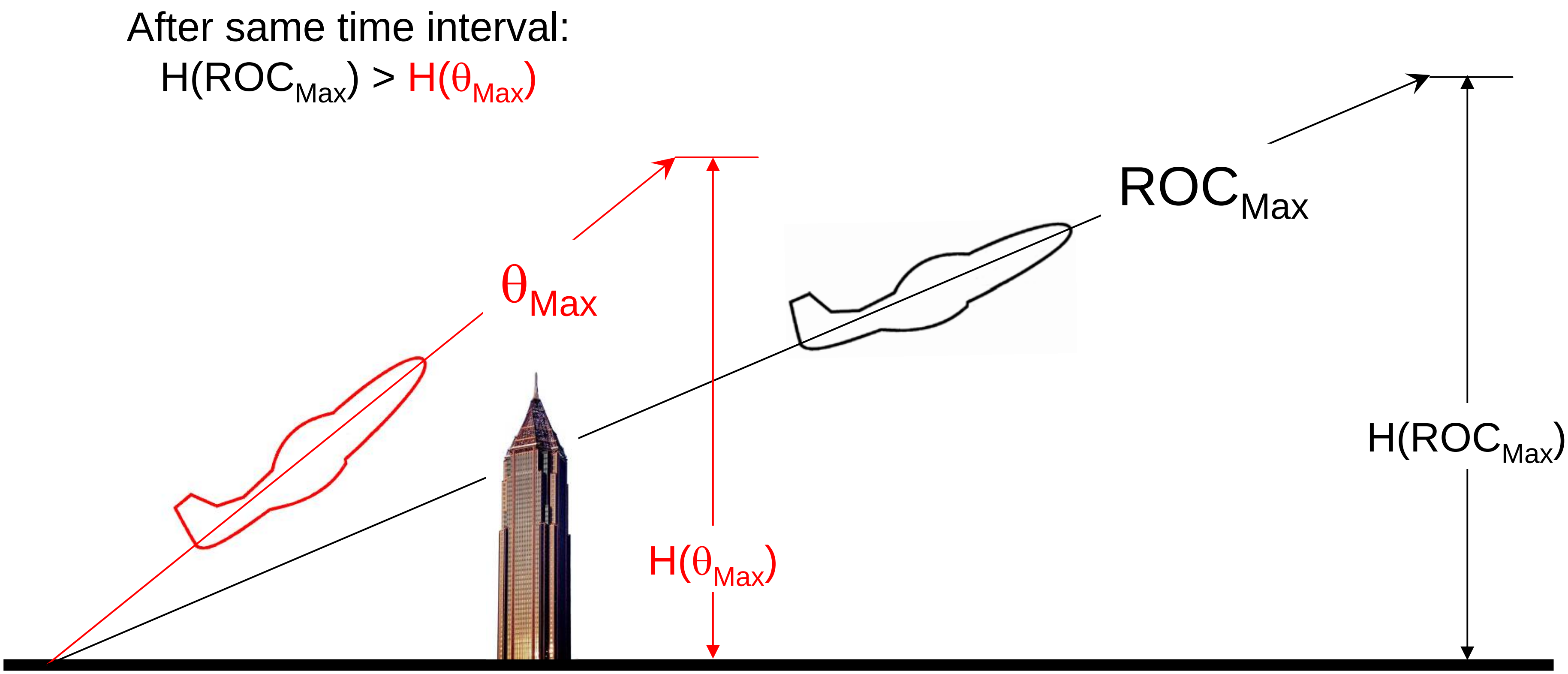
Rate of Climb



$$ROC = \frac{TV - DV}{W} = \frac{P_A - P_R}{W} = \frac{\text{Power Available} - \text{Power Required}}{W} = \frac{\text{Excess Power}}{W}$$

Rate of Climb

Here we have discussed the rate of climb, ROC. A related problem is the angle of climb, θ. Note that the maximum rate of climb, ROC_{Max} does not yield the maximum angle of climb, θ_{Max}. Pilots beware!!!



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Range

Range - How far can we fly?

In this discussion we will try develop integral equations to calculate the range of an airplane during its cruise leg.

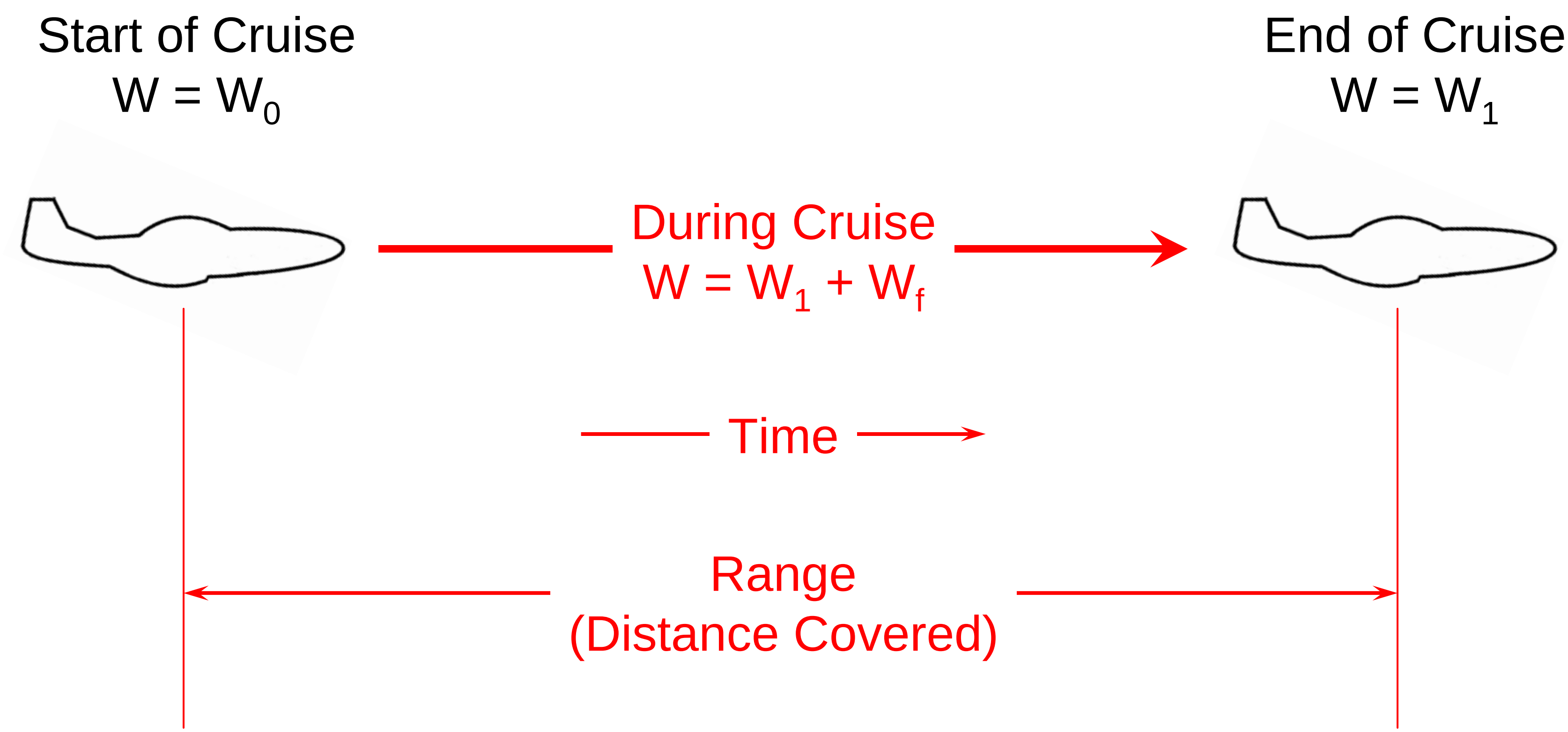
Range will be significantly affected by weight, so let us start by defining some weight quantities. Let:

- W_0 - Weight of the airplane at the start of cruise leg
- W_f - weight of fuel at a specific point in time; varies throughout the cruise leg
- W_1 - weight of the airplane at the end of the cruise leg
- W - weight of the airplane at a specific point in time; varies throughout the cruise leg

At any point during the cruise leg, then:

$$W = W_1 + W_f$$

Range



Note: No assumption is being made about the flight altitude at this time - altitude during cruise <u>may</u> or <u>may not</u> be constant!

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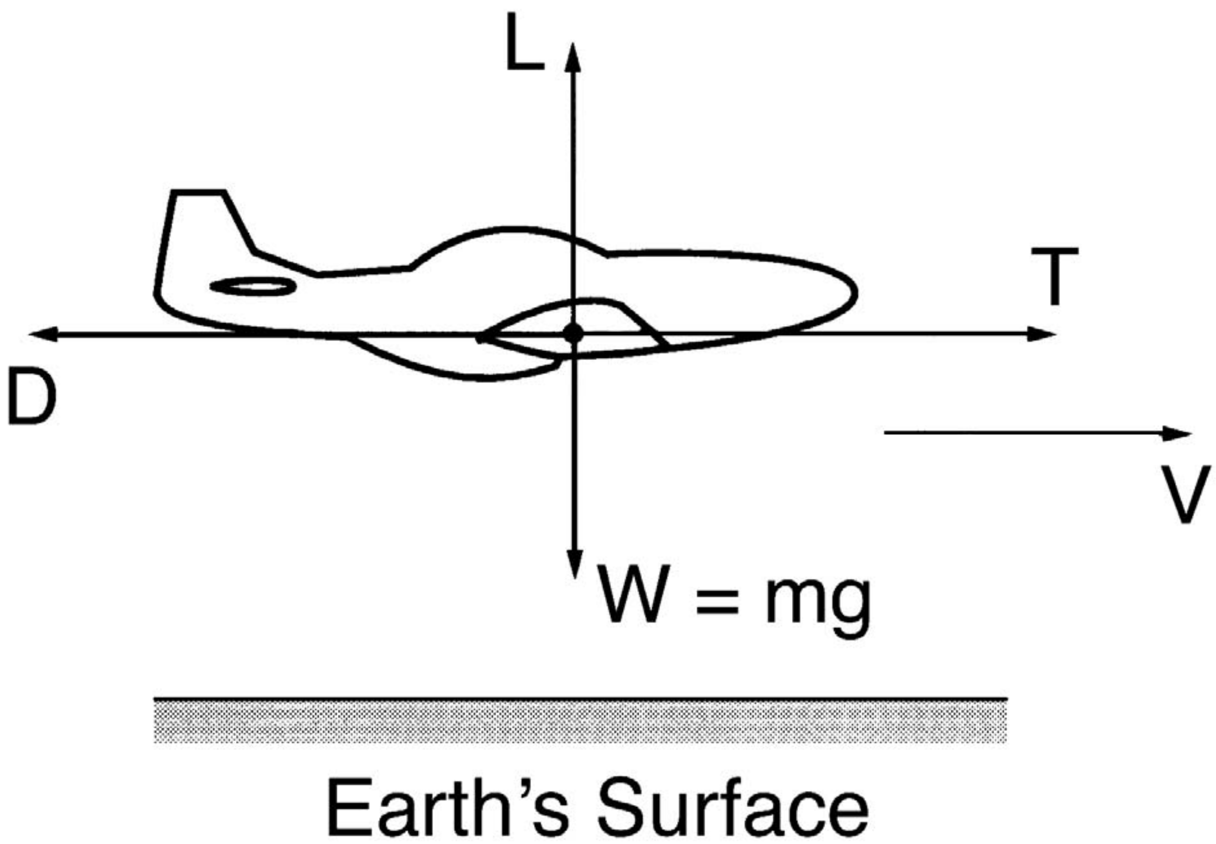
Range

Let us assume we are in level steady flight, and that our thrust vector is aligned with the airspeed:



$$T = D$$
$$L = W$$

Graphics source: Anderson, Aircraft Performance and Design

Range

After a certain amount of calculus and manipulation we can write the following integral equation for the range, R, of a turbojet airplane:

$$R = \int_{W_1}^{W_0} \frac{V}{c_t} \frac{L}{D} \frac{dW}{W} = \int_{W_1}^{W_0} \frac{1}{c_t} \sqrt{\frac{2}{\rho S}} \frac{C_L^{1/2}}{C_D} \frac{dW}{\sqrt{W}}$$

Very few assumptions were made to derive this equation:

- Flight in no-wind conditions
- Steady level flight

  $L = W$

  $T = D$
- Thrust aligned with V

Range

We can make some further assumptions and that will simplify the integral and allow us to evaluate it explicitly:

- The thrust specific fuel consumption, c_t is constant
- We are flying at a constant altitude, thus ρ is constant
- The wing area, S, is constant (we hope so!)
- We are flying at a constant value of $\left(C_L^{1/2}/C_D\right)$

These assumptions allow us to move various quantities in front of the integral:

Range

$$R = \frac{1}{c_t}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\int_{W_1}^{W_0}\frac{dW}{\sqrt{W}}$$

Now we can evaluate this integral explicitly:

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\left(W_0^{1/2} - W_1^{1/2}\right)$$

This is one version of the range equation for a <u>turbojet airplane.</u>

Range

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\left(W_0^{1/2} - W_1^{1/2}\right)$$

What is this equation telling us about obtaining maximum range for a turbojet airplane?

- For maximum range, fly at the value of C_L where $\left(C_L^{1/2}/C_D\right)$ is maximum.
- Fly at high altitude so that ρ is small (within limits…).
- Use an efficient engine with a low value of c_t.
- Carry lots of fuel.

Range

A similar equation can be derived for an airplane powered by a reciprocating engine/propeller combination:

$$R = \frac{\eta_{pr}}{c} \frac{C_L}{C_D} \ln \frac{W_0}{W_1}$$

Where,

η_{pr} is the propeller efficiency
c is the power specific fuel consumption

This is one of the best known equations in aeronautics. It is known as the Breguet Range Equation.

What is this equation telling us about obtaining maximum range for a reciprocating engine/propeller airplane?