

The Standard Atmosphere

AE 1350

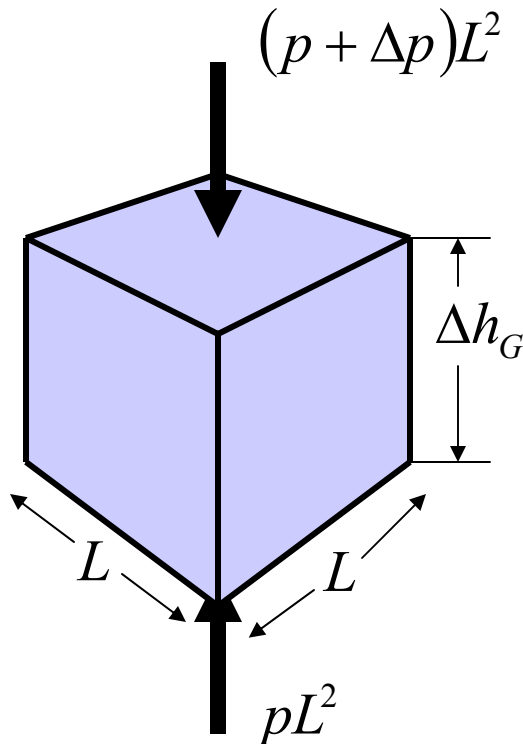


Weather and the Atmosphere

- Weather varies across location and time
- A standard atmosphere is defined for the Earth
 - We can say an airplane goes “100 mph” – and this will have *meaning*, since this implies we are talking about 100 mph in standard atmospheric conditions
 - Goes faster when air is less dense, has a tail wind, etc.
 - The standard atmosphere model is only a function of altitude (not latitude/longitude or time) and no wind
- In terms of quantities we’ve been using
 - Pressure (p) in pounds per square foot
 - Density (ρ) in slugs per cubic foot
 - Temperature (T) degrees Rankine, Fahrenheit, etc.

Hydrostatic Equation

- Standard atmosphere uses a model that accounts for pressure necessary to support the air above
- Consider a cube of air:

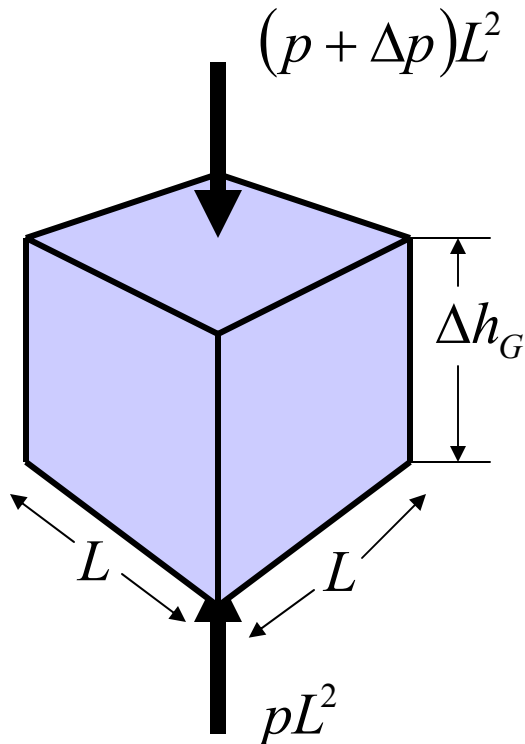


h is altitude

L is length/width of cube

Hydrostatic Equation

- Force must all balance
(from above and below plus gravity)



$$pL^2 = (p + \Delta p)L^2 + \rho g \Delta h_G L^2$$

g is acceleration due to gravity:

$$g = g_0 \left(\frac{r}{r + h_G} \right)^2$$

Gives us the hydrostatic equation:

go up and the pressure drops

$$\Delta p = -\rho g \Delta h_G$$

Geopotential Altitude

- The inverse squared gravity law makes the differential equations a mess
- Fix this by defining a new altitude so that gravity is effectively constant in the hydrostatic equation

$$\Delta p = -\rho g \Delta h_G \quad \text{becomes} \quad \Delta p = -\rho g_0 \Delta h$$

- To make this happen:

$$g \Delta h_G = g_0 \Delta h$$

$$\Delta h = \frac{g}{g_0} \Delta h_G$$

$$\Delta h = \frac{r^2}{(r + h_G)^2} \Delta h_G$$

So...

Both measures of altitude are nearly the same at low altitudes

Be careful which is being used!

Need to Relate Density and Pressure

- Model assumes air behaves like a perfect gas (intermolecular forces negligible)
- Equation of state:

$$p = \rho RT$$

where R is specific gas constant

$$R = 287 \frac{J}{kg \cdot ^\circ K}$$

$$R = 1716 \frac{ft \cdot lb}{slug \cdot ^\circ R}$$

Put it all together...

$$\Delta p = -\rho g_0 \Delta h$$

$$\frac{\Delta p}{p} = \frac{-\rho g_0 \Delta h}{p}$$

$$\frac{dp}{p} = \frac{-\rho g_0 dh}{\rho RT}$$

$$\boxed{\frac{dp}{p} = -\frac{g_0}{RT} dh}$$

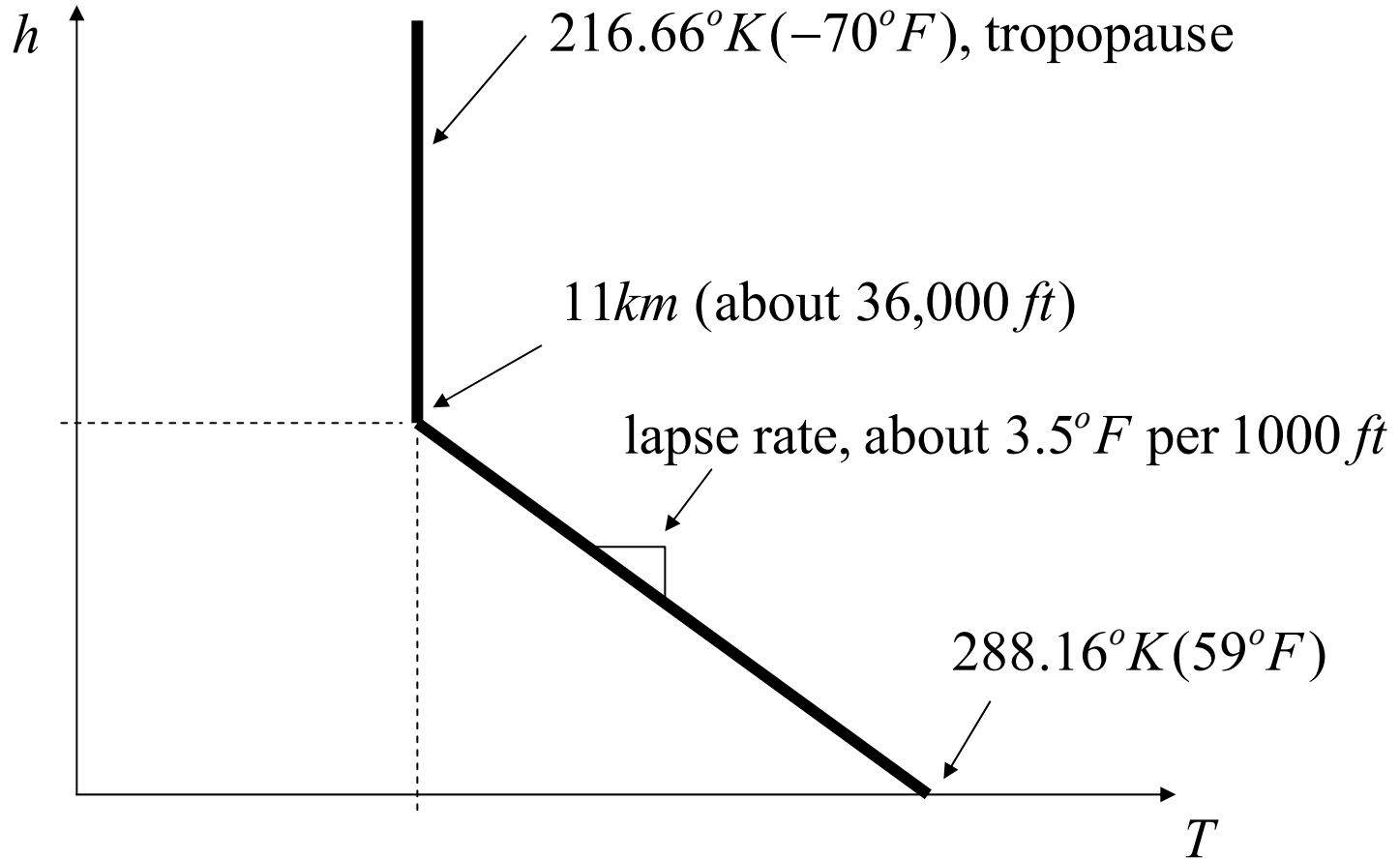
Now we have pressure as a function of temperature and altitude

The Standard Atmosphere Model

- The standard atmosphere is a temperature profile (function of altitude)
- Use $\frac{dp}{p} = -\frac{g_0}{RT} dh$ \longrightarrow pressure
- Use $\rho = \frac{p}{RT}$ \longrightarrow density
- Convert from geopotential altitude to regular altitude if needed

Temperature profile (first 20 km)

1976 U.S. Standard Atmosphere



In the Tropopause

T is a constant

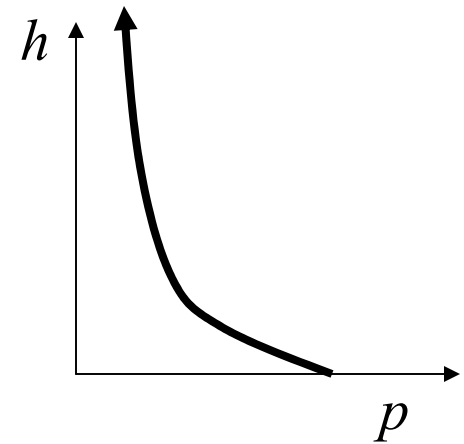
$$\frac{dp}{p} = -\frac{g_0}{RT} dh$$

$$\int_{p_1}^p \frac{dp}{p} = -\frac{g_0}{RT} \int_{h_1}^h dh$$

$$\ln \frac{p}{p_1} = -\frac{g_0}{RT} (h - h_1)$$

$$\frac{p}{p_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$

$$\frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$



Below the Tropopause

T is proportional to h , by lapse rate (a)

$$a = \frac{dT}{dh}$$

$$dh = \frac{dT}{a}$$

$$\frac{dp}{p} = -\frac{g_0}{RT} dh$$

$$\frac{dp}{p} = -\frac{g_0}{aR} \frac{dT}{T}$$

$$\int_{p_1}^p \frac{dp}{p} = -\frac{g_0}{aR} \int_{T_1}^T \frac{dT}{T}$$

$$\ln \frac{p}{p_1} = -\frac{g_0}{aR} \ln \frac{T}{T_1}$$

$$\frac{p}{p_1} = \left(\frac{T}{T_1} \right)^{-\frac{g_0}{aR}}$$

$$\frac{p}{p_1} = \frac{\rho RT}{\rho_1 RT_1} = \left(\frac{T}{T_1} \right)^{-\frac{g_0}{aR}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1} \right)^{-\left(\frac{g_0}{aR} + 1 \right)}$$
$$T = T_1 + a(h - h_1)$$

Other Constants Needed

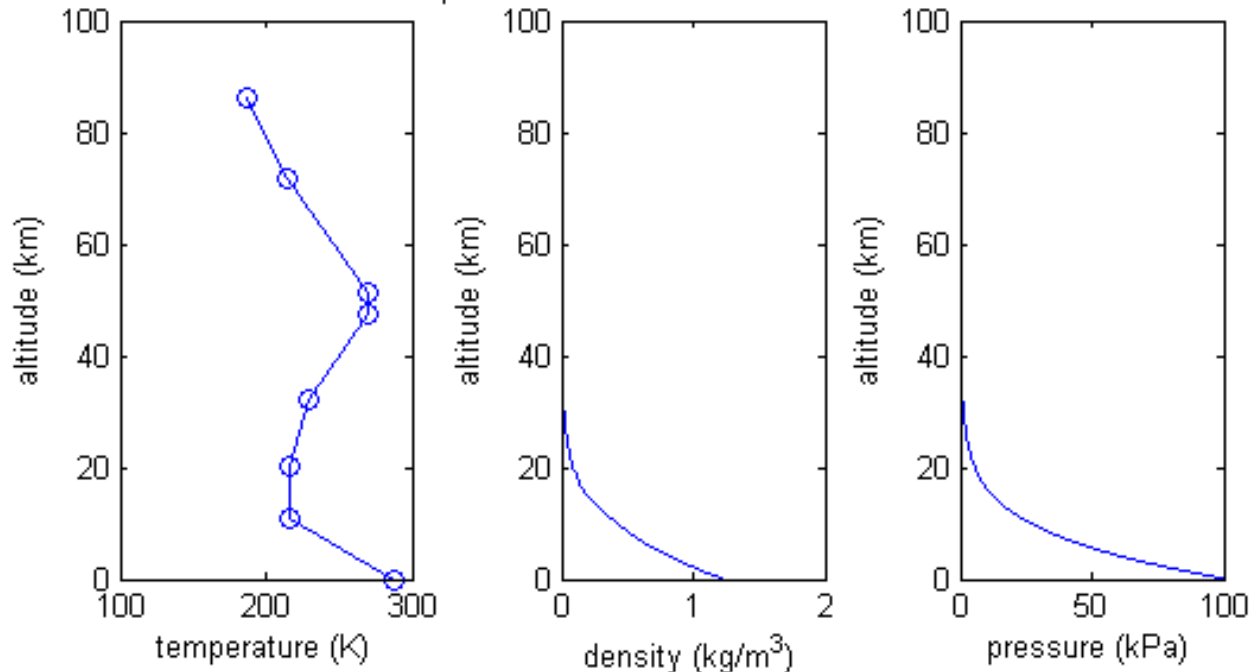
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- To start the integration, need values at $h = 0$

$$p(0) = 101325 \frac{N}{m^2} = 2112.2 \frac{lb}{ft^2}$$

$$\rho(0) = 1.225 \frac{kg}{m^3} = 0.002377 \frac{slug}{ft^3}$$

The 1976 U.S. Standard Atmosphere



from wikipedia

More Useful “Altitudes”

- There are useful metrics to compare current atmospheric conditions to the standard atmosphere

- Pressure altitude: Altitude in standard atmosphere with pressure corresponding to current pressure
- Density altitude: Altitude in standard atmosphere with density corresponding to current density
- Temperature altitude: ...you get it...

1976 U.S. Standard Atmosphere is in your text, Appendix A & B

