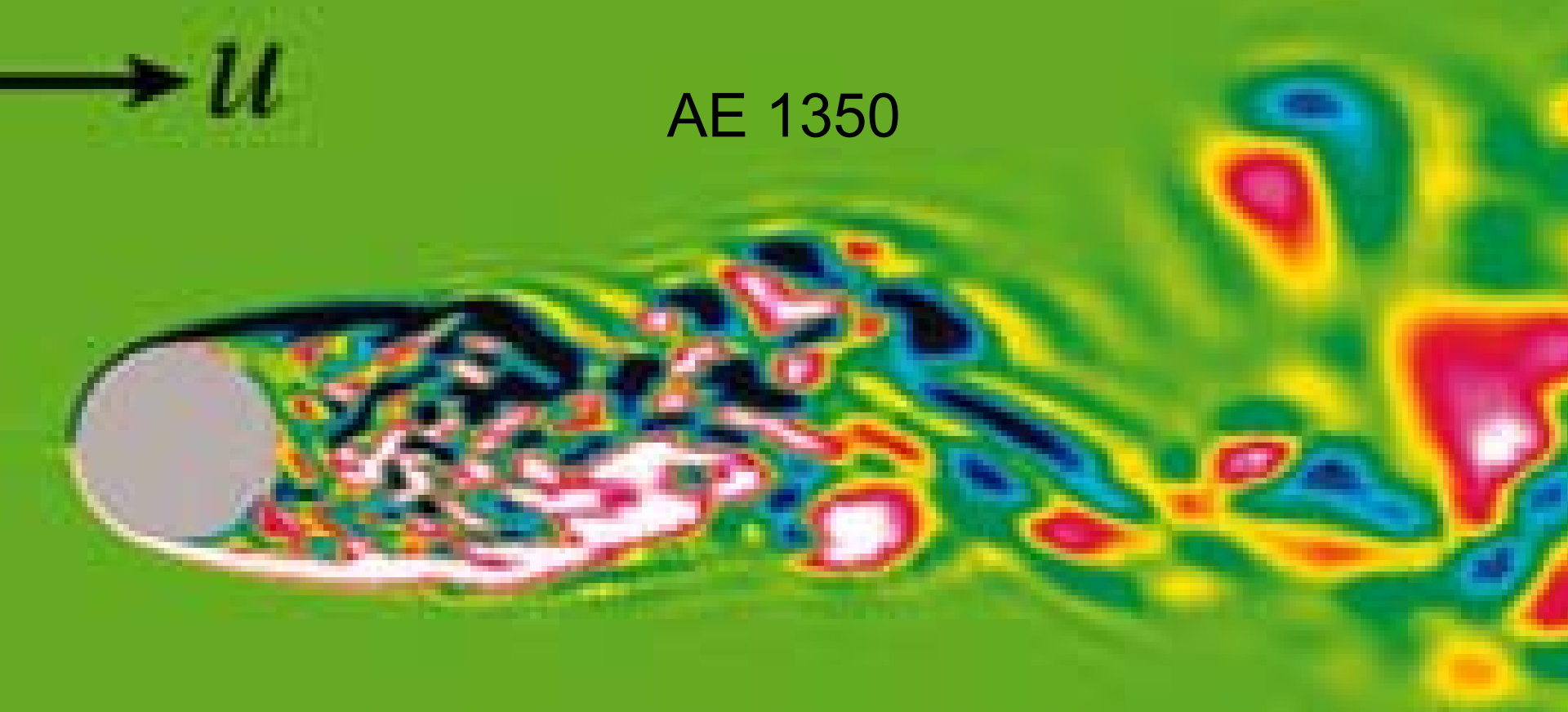


# Aerodynamics Introduction



# Revisiting Flow Characterization Quantities

- Mach number
- Reynolds number
- Angle of attack

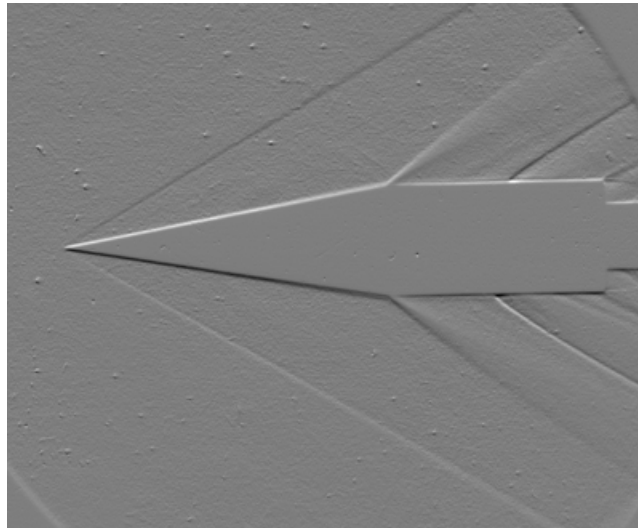
# Incompressible vs. Compressible

## Function of Mach number

- Air is an example of a compressible fluid
  - Its density changes if temperature changes, or if some external force is applied
- A flow is said to be incompressible if there are no changes in density attributable to (or caused by) the velocity or speed of the flow
- Theory and observations in wind tunnels suggest that most flows may be treated as incompressible (i.e., constant density) until the Mach number is sufficiently high (>0.4 or so)
- $M = \text{Velocity} / \text{Speed of Sound}$

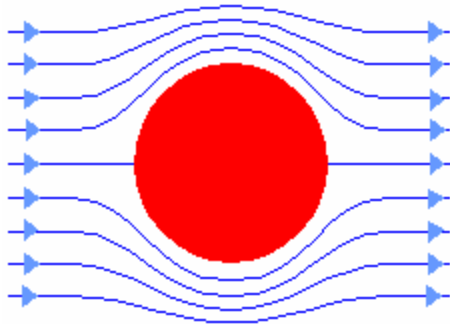
# Why of function of M?

- If there is sufficient time for the sound waves to travel before the fluid particle arrives, the fluid particles downstream will “hear” the message and tend to get out of the way
- Otherwise, there will be a crush (compression), or even a large jump in density (shock wave)

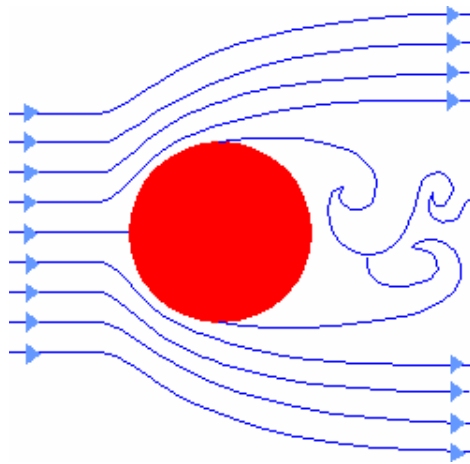


# Viscid vs. Inviscid Flow

## Function of Reynolds Number ( $R_n$ )



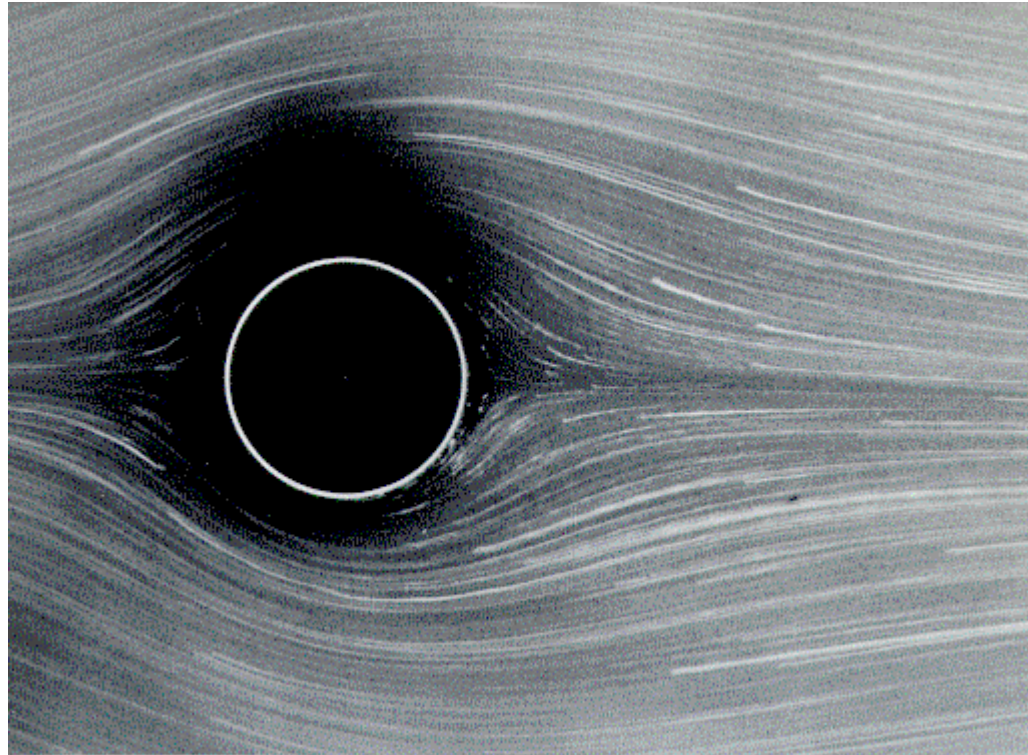
Inviscid (ideal) flow



Viscous flow

- Streamlines describe the path the fluid particles will take
- Flow velocity is tangential to the streamline
- Viscosity alters the shape of streamlines around bluff bodies
- Scientists inject smoke particles into streamlines to make them visible to the naked eye

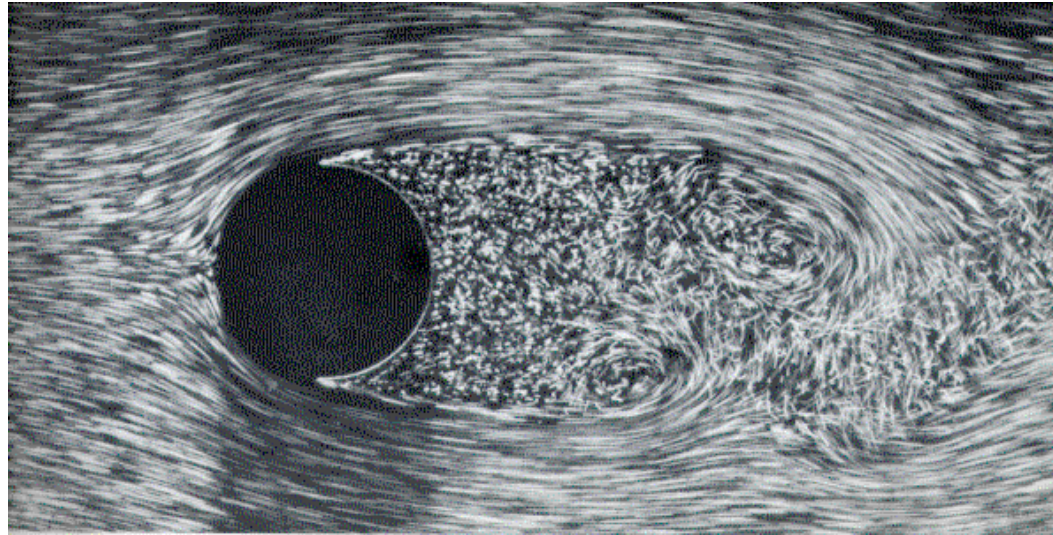
# Cylinder at Low Reynolds Number (10)



$$\text{Reynolds Number} = \frac{\rho V D}{\mu}$$

*where* D = Cylinder dia.

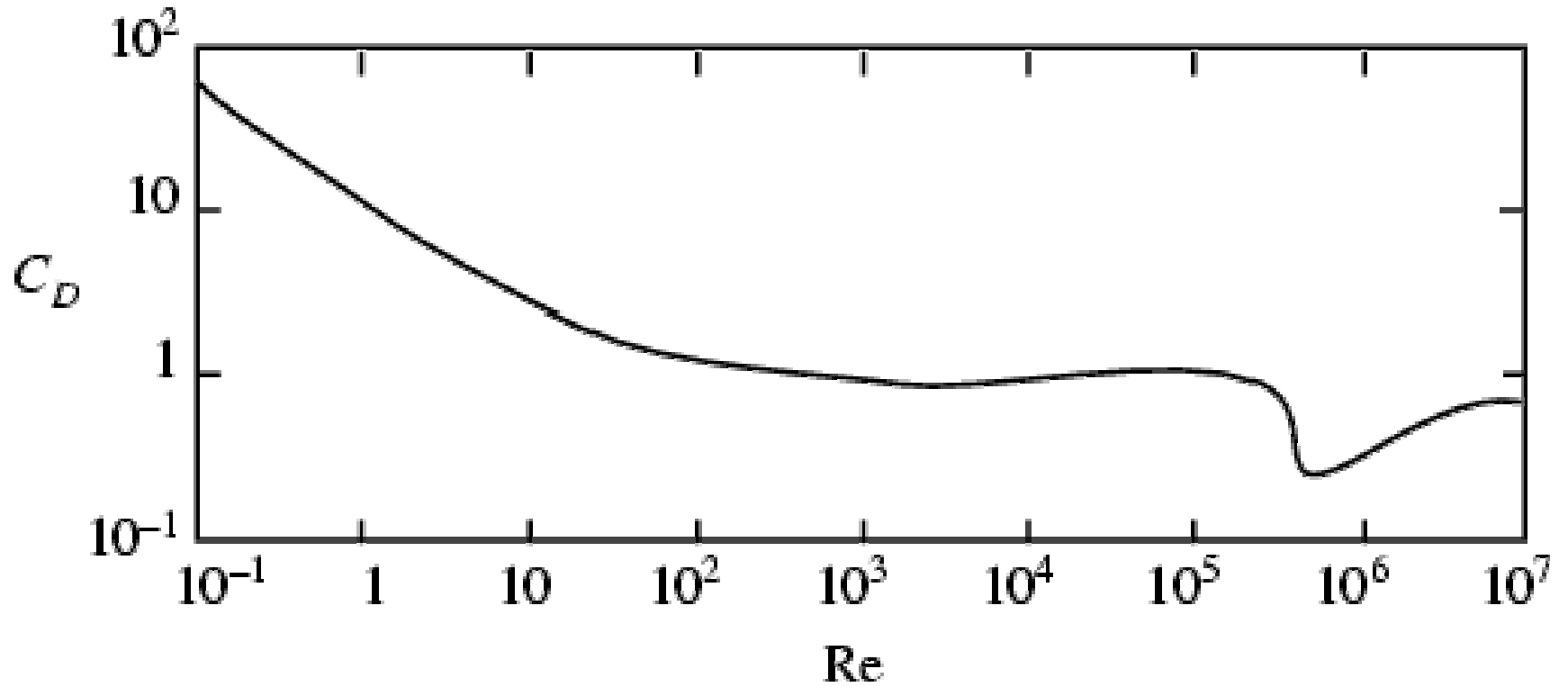
# Cylinder at “High” Reynolds Number (2000)



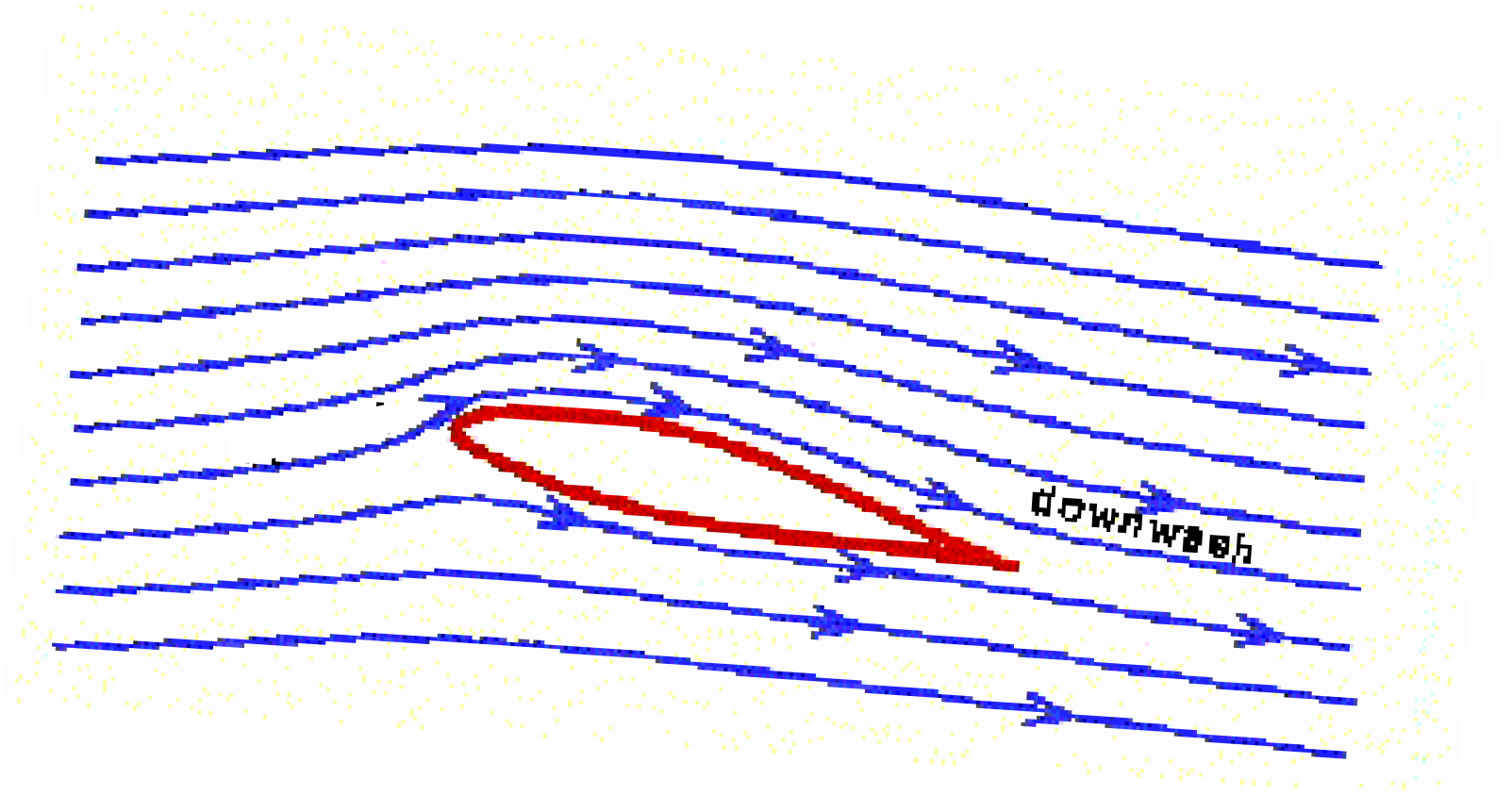
$$\text{Reynolds Number} = \frac{\rho V D}{\mu}$$

*where* D = Cylinder dia.

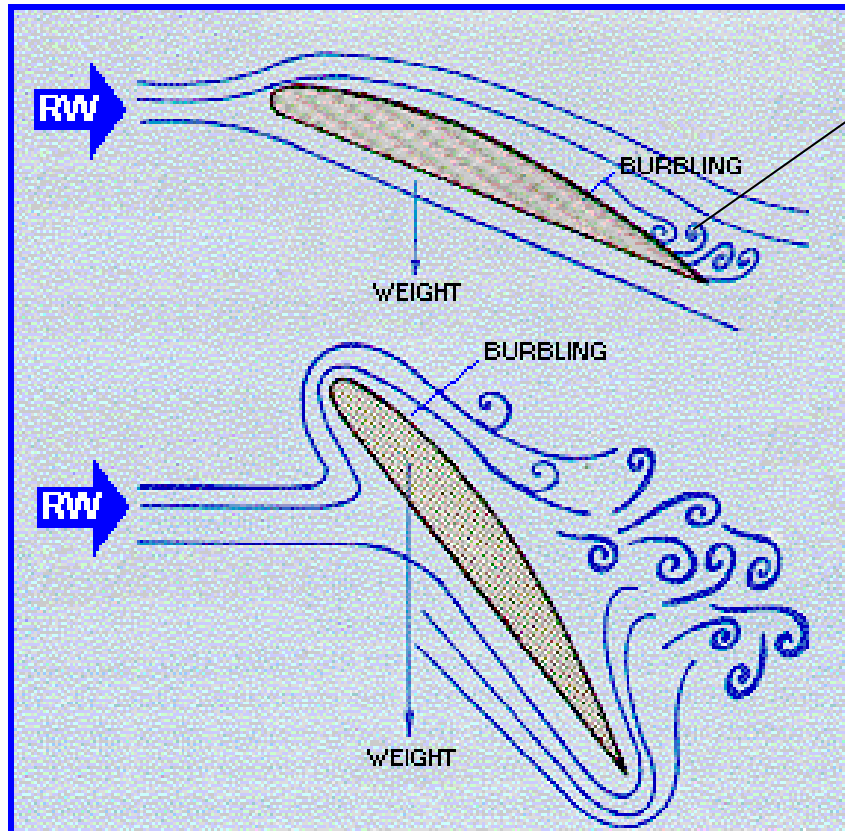
# Drag Coefficient on a Cylinder



# Streamlines over an Airfoil



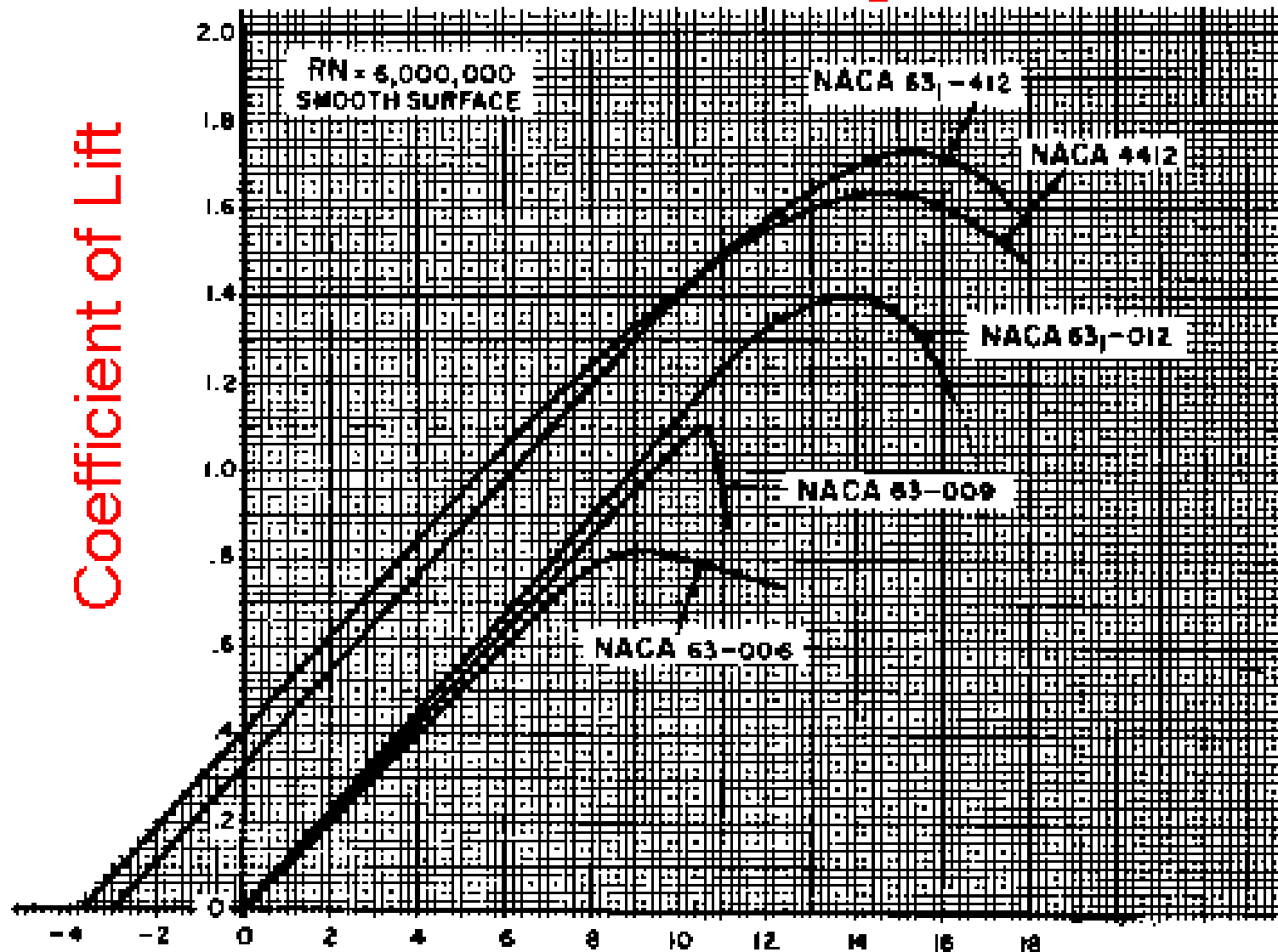
# High Angle of Attack (High angle with respect to flow)



Flow  
Separation

Figure 5-8 Airfoil approaching and entering a stall

# Coefficient of Lift vs Angle of Attack



Coefficient of Lift

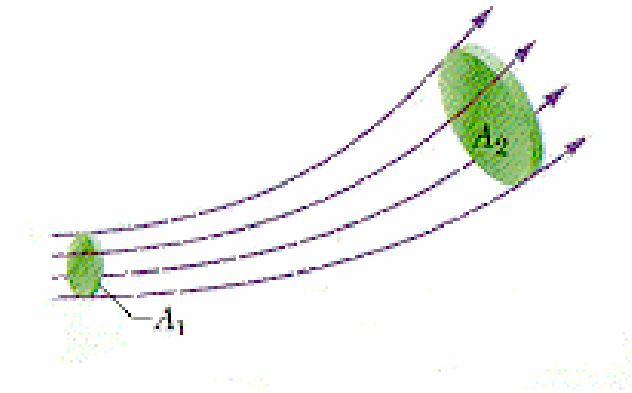
Angle of Attack

# Flow Physics

- Continuity
- Conservation of Momentum
- Conservation of Energy

# Continuity

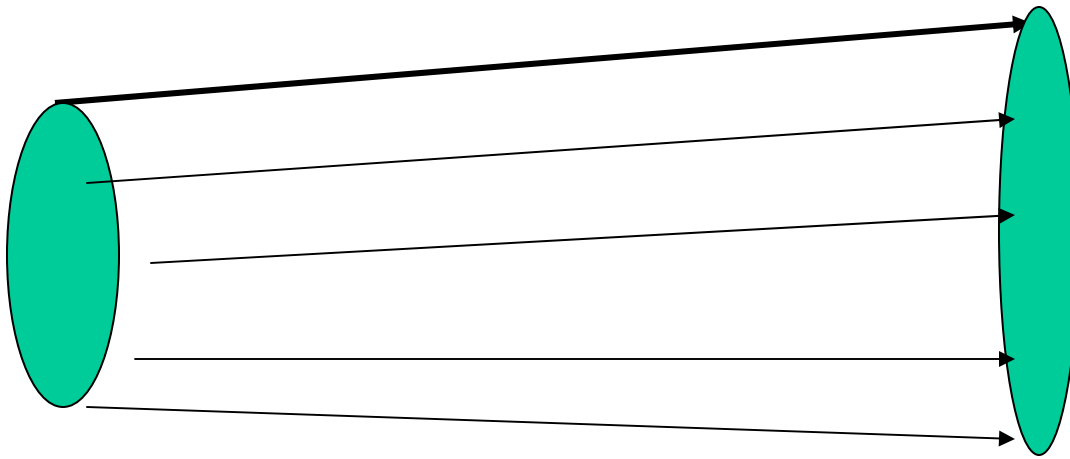
- Consider a stream tube, i.e. a collection of streamlines that form a tube-like shape
- Within this tube mass can not be created or destroyed
- The mass that enters the stream tube from the left (e.g. at the rate of 1 kg/sec) must leave on the right at the same rate (1 kg/sec)



# Continuity Equation

Rate at which mass leaves =  $\rho_2 A_2 V_2$

Rate at which mass enters =  $\rho_1 A_1 V_1$



Area  $A_1$   
Density  $\rho_1$   
Velocity  $V_1$

Area  $A_2$   
Density  $\rho_2$   
Velocity  $V_2$

# Continuity Equation

## 2 Versions

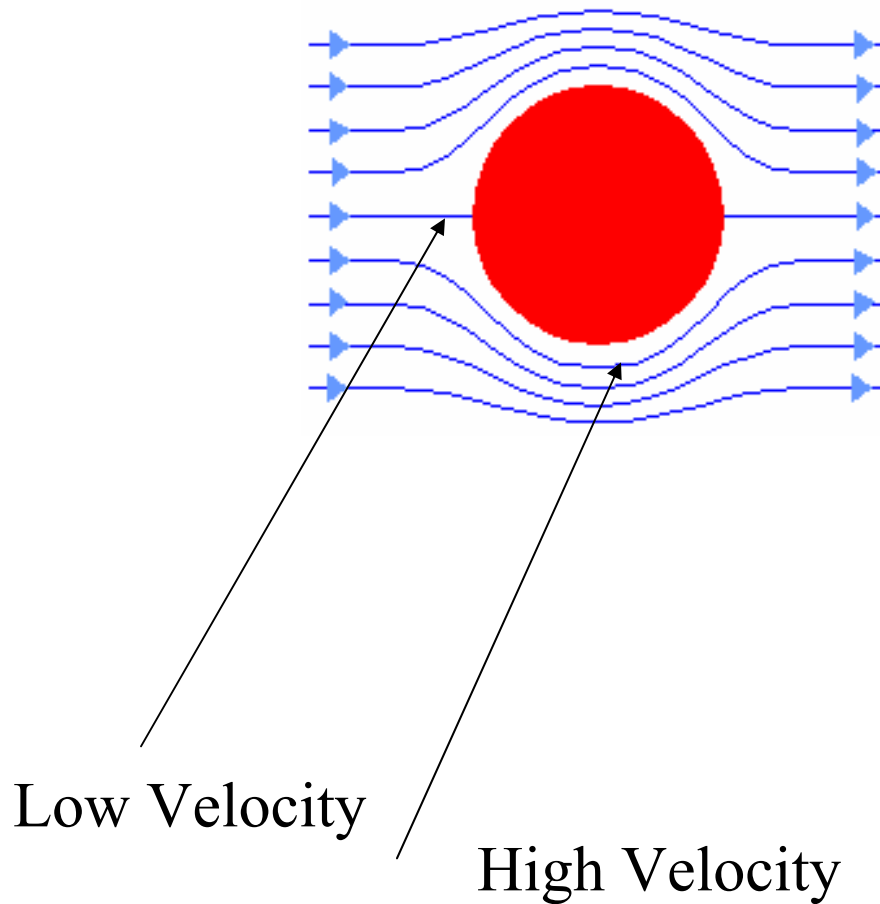
In compressible flow through a “tube”

$$\rho AV = \text{constant}$$

In incompressible flow,  $\rho$  does not change. Thus,

$$AV = \text{constant}$$

# More on Continuity

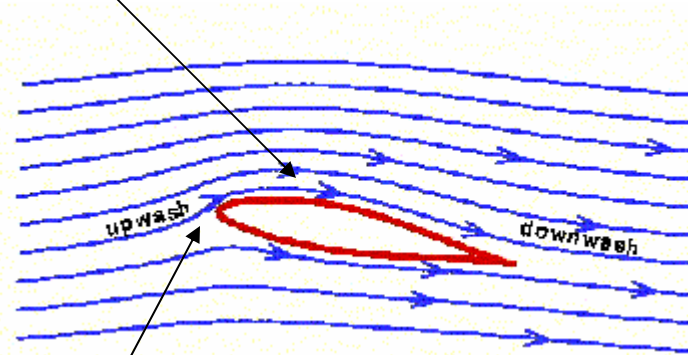


$$AV = \text{constant}$$

If Area between streamlines is high, the velocity is low and vice versa.

# More on Continuity

High Velocity



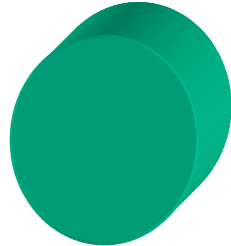
$$AV = \text{constant}$$

If Area between streamlines is high, the velocity is low and vice versa

In regions where the streamlines squeeze together, velocity is high

Low Velocity

# Conservation of Momentum

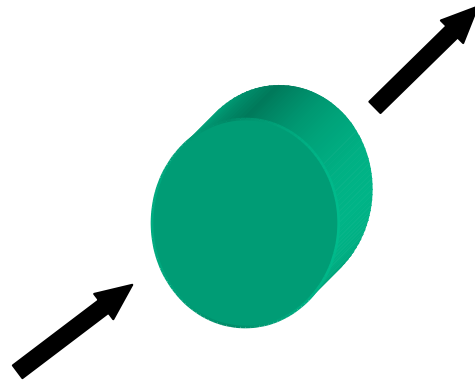
- Newton's 2<sup>nd</sup> Law
  - Rate of change of momentum is sum of forces, i.e., “ $F = ma$ ”
- Consider a small slice of stream tube 
- Rate of change of momentum of the fluid particles within this stream tube slice must be due to forces acting on it
  - Force = time rate of change of Momentum

# Momentum Equation: Rate of Change of Momentum

Mass Flow Rate in = Mass Flow rate out (Continuity)

$$\rho VA = (\rho+d\rho)(V+dV)(A+dA)$$

Density  $\rho$   
velocity  $V$   
Area =  $A$



Density  $\rho+d\rho$   
velocity  $V+dV$   
Area =  $A+dA$

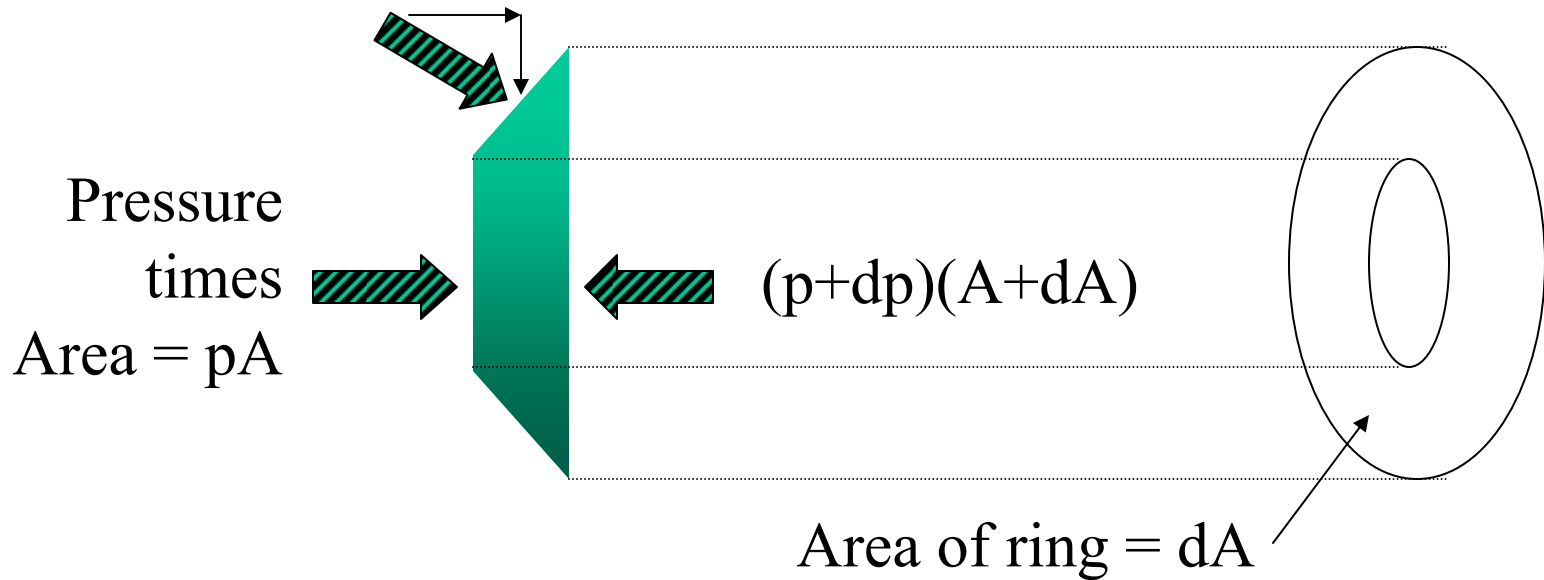
Momentum rate in =  
Mass flow rate times velocity  
=  $\rho V^2 A$

Momentum Rate out =  
Mass flow rate times velocity  
=  $\rho VA (V+dV)$

Rate of change of momentum within this element =  
Momentum rate out - Momentum rate in  
=  $\rho VA (V+dV) - \rho V^2 A = \rho VA dV$

# Momentum Equation: Forces Acting on Stream Tube

Horizontal Force = Pressure times area of the ring =  $(p+dp/2)dA$



$$\begin{aligned} \text{Net force} &= pA + (p+dp/2)dA - (p+dp)(A+dA) \\ &= -Adp - dp \cdot dA/2 \approx -Adp \\ &\quad \uparrow \\ &\quad \text{Product of two} \\ &\quad \text{small numbers} \end{aligned}$$

# Momentum Equation

- Rate of change of momentum =  $\rho AVdV$
- Forces acting on the stream tube =  $-Adp$ 
  - Note: We have neglected all other forces: viscous, gravity, electrical and magnetic forces
- Equating the two factors (and divide by A), we get  
Euler's equation

$$\rho VdV + dp = 0$$

# Momentum Equation for Incompressible Flow

- For incompressible flows ( $\rho$  constant), Euler's equation can be integrated:

$$\rho V dV + dp = 0$$

$$\rho \int V dV + \int dp = 0$$

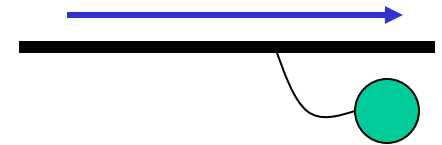
$$\frac{1}{2} \rho V^2 + p = \text{Const}$$

- ...called Bernoulli's Equation (we've seen before!)
- This “stagnation pressure” is constant along a streamline, High Reynolds number, low Mach number

# Pitot Tube: Measure Airspeed using Bernoulli's Equation

- Measure local pressure of air:

$p_S$ , “static pressure”



- Measure pressure after bringing to zero speed relative to instrument (i.e., bring to stagnation):

$p_T$ , “total pressure”



- From Bernoulli:

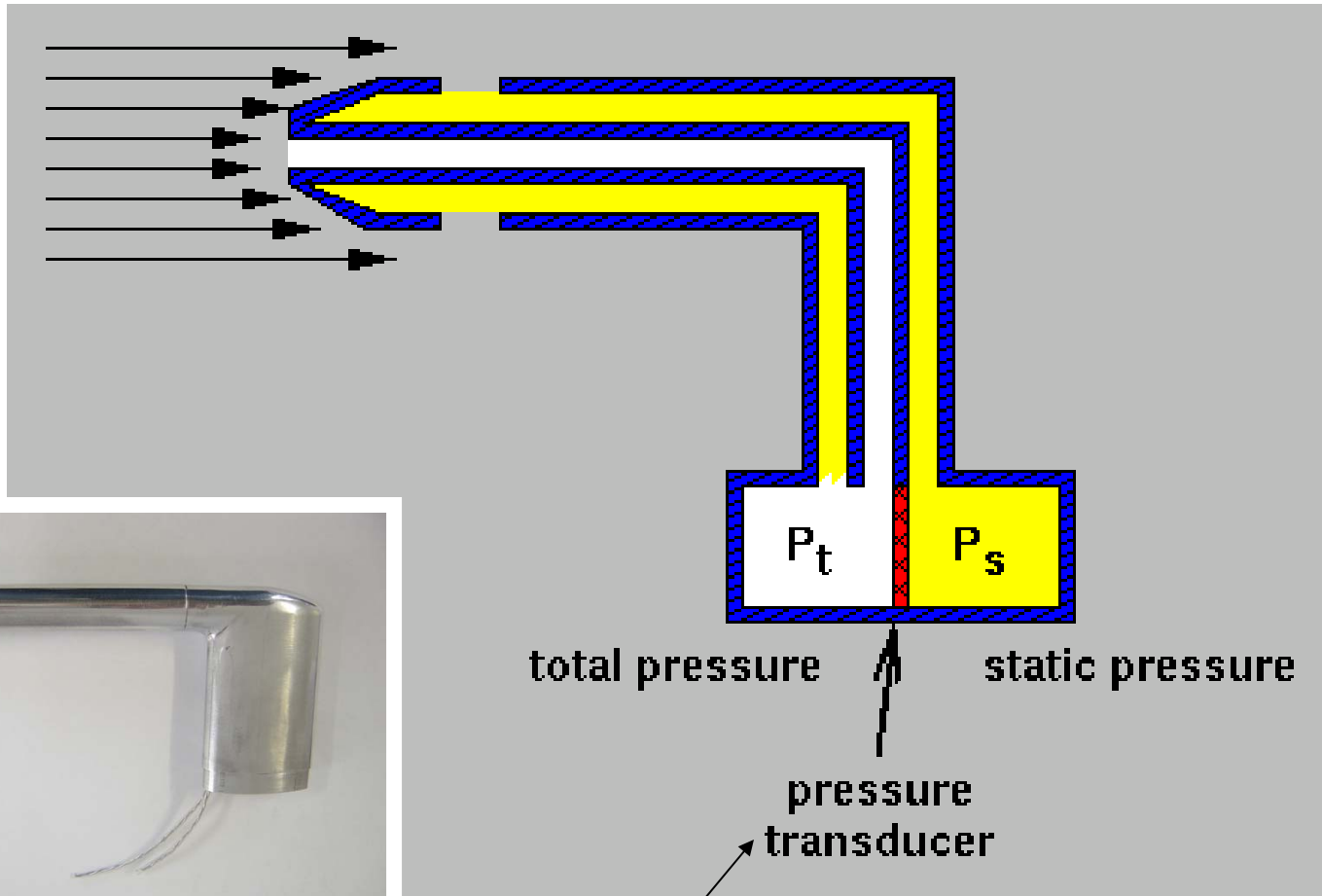
$$\frac{1}{2}\rho V^2 + p_S = p_T$$

- Determine velocity of flow with respect to instrument:

$$V = \sqrt{\frac{2(p_T - p_S)}{\rho}}$$

- Note: really only need difference between  $p_S$  and  $p_T$

# Pitot Tube



Measure pressure difference

$$V = \sqrt{\frac{2(p_T - p_S)}{\rho}}$$

(Wikipedia)

# Pitot-static system



- Total pressure probe(s) not behind propulsion sources, near front of aircraft – usually on sides of fuselage near front or front of a wing
- Static pressure port(s) in location with minimum disturbance from aircraft itself – usually sides of fuselage
- Airspeed indicator shows measured airspeed to pilot

2 on each side of this Boeing 757

# Airspeed Indicator

- Mechanically uses Bernoulli's equation to show airspeed – not corrected for density
- Aircraft are typically flown via this so-called indicated airspeed (not corrected for density, mechanical errors, etc.) in knots

$$V = \sqrt{\frac{2(p_T - p_S)}{\rho_{\text{standard}}}}$$

