Neural Network Adaptive Control of Systems with Input Saturation

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Abstract

In the application of adaptive flight control, significant issues arise due to limitations on the plant inputs, such as actuator displacement limits. The concept of utilizing a modified reference model to prevent an adaptation law from "seeing" this system-input characteristic is described. The method allows correct adaptation while the plant input is saturated. To apply the method, estimates of actuator positions must be found. However, the adaptation law can correct for errors in these estimates. A theorem of boundedness for all system signals is included for a single hidden layer neural network adaptive law. The domain of attraction is also discussed.

1. Background

In recent years, several theoretical developments have given rise to the use of artificial Neural Networks (NNs) for adaptive control of nonlinear systems[1,2]. The use of NN adaptive flight control has been demonstrated in piloted hardware-in-the-loop simulation and flight test on the X-36 aircraft[34]. This approach has been utilized to enable a single controller to handle multiple versions of guided munitions[5], and to replace traditional reusable launch vehicle gain tables[6].

Input saturation implies that either a position or rate limit has been exceeded. Input saturation presents a significant problem for adaptive control, because in causes an adaptation law to be “tricked” by unexpected effects - a behavior analogous to integrator windup in a linear controller. However, unlike an integrator designed for a selected linear response, it may be possible for an adaptive law to function properly (as designed) during finite periods of input saturation.

One approach used is to avoid saturation altogether by either command or feedback signal adjustment. This has been demonstrated in an adaptive control setting[7,8,9]. A second approach involves slowing or halting adaptation as saturation is entered. A common *ad-hoc* approach for most adaptive control methods is to simply stop adaptation completely when any input saturates.

Another approach to the problem of adaptive control with input displacement saturation is augmenting the error signal. These methods apply to Model Reference Adaptive Control (MRAC), with an early result given by[10] without a stability proof. For this method, the effect of the saturation nonlinearity on the reference model tracking error is removed by adding a signal derived from the actual plant input to the error signal[7, 11,12,13].

The method described below was originally motivated by the application in[6], and is most closely related to the method in[10], except that it relies on a modification to the reference model. As a result, it is not limited to saturation nonlinearities, linear plants, or linear reference models. The method described is also similar to Anti-Windup Bumpless Transfer (AWBT) theory for non-adaptive controllers, specifically the Hanus conditioning technique[14] which also includes the concept of a miss-match between commanded and actual plant input.

2. Pseudo-Control Hedging Architecture

The method described here is termed Pseudo-Control Hedging (PCH). The purpose of the method is to prevent the adaptive element of an adaptive control system from adapting to selected plant input characteristics. The specific case of PCH applied to an adaptive control architecture that includes an approximate dynamic inversion is illustrated in Figure 1.

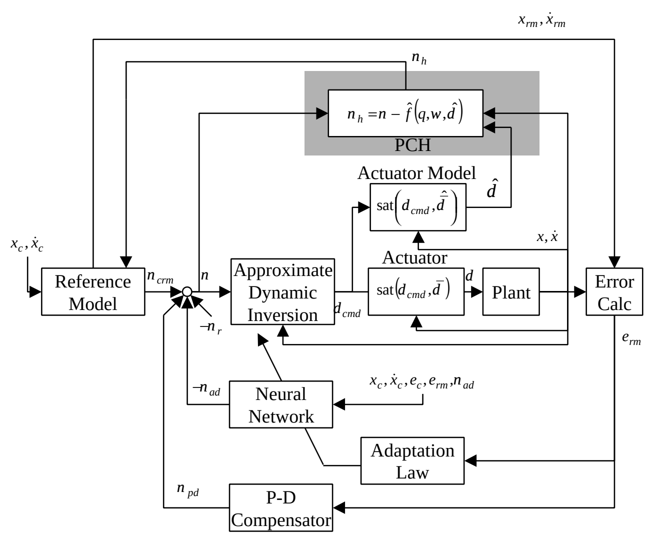

Figure 1 – MRAC including an approximate dynamic inversion with PCH compensation

Consider n-degree-of-freedom plant dynamics of the form

$$\ddot{x} = f(x, \dot{x}, d) \tag{1}$$

where $x, \dot{x} \in \Re^n, d \in \Re^m$ with $m \geq n$. An approximate dynamic inversion element is developed to determine actuator commands of the form

$$d_{cmd} = \hat{f}^{-1}(x, \dot{x}, n) \tag{2}$$

where n is the "pseudo-control", and represents a desired $\ddot{x}$ that is expected to be approximately achieved by d_{cmd}. That is, this dynamic inversion element is designed without consideration of actuator saturation. This command (d_{cmd}) will not necessarily equal the actual control (d) due to saturation.

To get the PCH signal (n_h), an estimated actuator position ($\hat{d}$) is determined based on a model or measurement. This estimate is then used to get the difference between commanded pseudo-control and the estimated-actual pseudo-control

$$\begin{aligned} n_h &= \hat{f}\left(x,\dot{x},d_{cmd}\right)-\hat{f}\left(x,\dot{x},\hat{d}\right) \\ &= n-\hat{n} \end{aligned} \tag{3}$$

The reference model update is

$$\ddot{x}_{rm} = n_{crm}\left(x_{rm},\dot{x}_{rm},x_c,\dot{x}_c\right)-n_h\,. \tag{4}$$

where $x_c, \dot{x}_c$ are the external commands. This choice of reference model update will remove the actuator characteristic from reference model tracking error, discussed below. The instantaneous pseudo-control output of the reference model in the feed-forward path is n_{crm}.

2.1. Reference Model Tracking Error Dynamics

The complete pseudo-control signal for the system is

$$n = n_{crm} + n_{pd} - n_{ad} - n_r \tag{5}$$

Note that the PCH signal (n_h) does not appear here, and is only used to modify the reference model in Eqn 4. The second term of the right-hand-side (n_{pd}) is the output of proportional-derivative compensation acting on the reference model tracking error

$$n_{pd} = \begin{bmatrix} K_p & K_d \end{bmatrix} e_{rm}\,, \tag{6}$$

where $K_d > 0, \in \Re^{n\times n}$ and $K_p > 0, \in \Re^{n\times n}$ are diagonal matrices containing desired second-order linear reference model error dynamics and $e_{rm}^T = \begin{bmatrix} (x_{rm}-x)^T & (\dot{x}_{rm}-\dot{x})^T \end{bmatrix}$, chosen such that

$$A = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix} \tag{7}$$

is Hurwitz. The robustifying signal (n_r) is chosen as

$$n_r = -\left(\|Z\| + \bar{Z}\right)K_r r^T \tag{8}$$

with $K_r \in \Re^{n\times n}$ diagonal, $K_r > 0$. The r signal is a linear combination of the reference model tracking error components given by $r = e_{rm}^T PB$ where $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$ and $P \in \Re^{2n\times 2n}$ is the positive definite solution to the Lyapunov equation $A^T P + PA + Q = 0$.

The model tracking error dynamics are now found by differentiating e_{rm} (temporarily ignoring n_r for clarity):

$$\dot{e}_{rm} = Ae_{rm} + B\left[n_{ad}\left(x,\dot{x},\hat{d}\right) - f\left(x,\dot{x},d\right) + \hat{f}\left(x,\dot{x},\hat{d}\right)\right] \tag{9}$$

When one assumes that d is exactly known ($\hat{d} = d$), it follows from Eqn 9 that

$$\dot{e}_{rm} = Ae_{rm} + B\left[n_{ad}\left(x,\dot{x},d\right) - \Delta\left(x,\dot{x},d\right)\right] \tag{10}$$

where $\Delta(x,\dot{x},d) = f(x,\dot{x},d) - \hat{f}(x,\dot{x},d)$ is regarded as model error to be approximated by n_{ad}. Eqn 10 is of the same form as the model tracking error dynamics seen in previous work, regardless of saturation. These dynamics can form the basis of an adaptive law, where selected actuator characteristics have now been removed.

Remark 1: When one returns to the assumption made to arrive at Eqn 10 above, and makes the less restrictive assumption that one can express $d = d\left(\hat{d}\right)$, it follows from Eqn 9 that

$$\dot{e}_{rm} = Ae_{rm} + B\left[n_{ad}\left(x,\dot{x},\hat{d}\right) - \Delta\left(x,\dot{x},\hat{d}\right)\right] \tag{11}$$

where $\Delta\left(x,\dot{x},\hat{d}\right) = f\left(x,\dot{x},d\left(\hat{d}\right)\right) - \hat{f}\left(x,\dot{x},\hat{d}\right)$ appears as model error to the adaptive law.

Input saturation can be modeled with

$$d_i = \mathrm{sat}\left(d_{cmd_i},\bar{d}_i\right) \equiv \begin{cases} d_{cmd_i}, & \left|d_{cmd_i}\right| \le \bar{d}_i \\ \bar{d}_i \dfrac{d_{cmd_i}}{\left|d_{cmd_i}\right|}, & \left|d_{cmd_i}\right| > \bar{d}_i \end{cases} \tag{12}$$

$\forall i = 1,2,\ldots,m$, or in corresponding vector notation $d = \mathrm{sat}\left(d_{cmd},\bar{d}\right)$ with an actuator model being $\hat{d} = \mathrm{sat}\left(d_{cmd},\hat{\bar{d}}\right)$. Note that the hedge signal (n_h) will be zero when no actuator *model* element is at a limit.

Remark 2: When the actuator and actuator model are "ideal" (the actual position, estimated position, and commanded position of the actuator are equal) the addition of PCH has no effect on any system signal.

2.2. A Neural Network as the Adaptive Element

Single Hidden Layer (SHL) Perceptron NNs are universal approximators in that they can approximate any smooth nonlinear function to within arbitrary accuracy, given a sufficient number of hidden layer neurons and input information[15]. Here, a SHL NN is trained online to cancel model error with feedback. The input-output map can be expressed as

$$n_{ad_k} = b_w q_{w,k} + \sum_{j=1}^{n_2} w_{j,k}\, s_j\left(b_v q_{v,j} + \sum_{i=1}^{n_1} v_{i,j} x_{in_i}\right) \tag{13}$$

where $k = 1,\cdots,n$. Here n_1, n_2, and n are the number of inputs, hidden layer neurons, and outputs respectively; and $x_{in_i}, i = 1,2,\ldots,n_1+1$ contains the NN inputs. The scalar function s_j is a sigmoidal activation function that represents the "firing" characteristics of the neuron, or

$$s_j(z) = \frac{1}{1+e^{-a_j z}} . \qquad (14)$$

The factor a_j is the activation potential, and is a distinct value for each hidden-layer neuron ($j = 1,2,...,n_2$).

For convenience define the two weight matrices

$$V = \begin{bmatrix} q_{v,1} & \cdots & q_{v,n_2} \\ v_{1,1} & \cdots & v_{1,n_2} \\ \vdots & \ddots & \vdots \\ v_{n_1,1} & \cdots & v_{n_1,n_2} \end{bmatrix}, \; W = \begin{bmatrix} q_{w,1} & \cdots & q_{w,n} \\ w_{1,1} & \cdots & w_{1,n} \\ \vdots & \ddots & \vdots \\ w_{n_2,1} & \cdots & w_{n_2,n} \end{bmatrix}, \qquad (15)$$

and $Z = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix}$; and define a sigmoid vector as

$$s(z) = \begin{bmatrix} b_w & s(z_1) & s(z_2) & \cdots & s(z_{n_2}) \end{bmatrix}^T \qquad (16)$$

where $b_w > 0$ allows for the threshold q_w to be included in the weight matrix W, $z = V^T \bar{x}$ and $\bar{x}^T = \begin{bmatrix} b_v & x_{in}^T \end{bmatrix}$ where $b_v > 0$ is an input bias that allows for the threshold q_v to be included in the weight matrix V.

With the above definitions, the input-output map of the SHL NN can be written in a matrix form as

$$n_{ad}(W, V, \bar{x}) = W^T s\left(V^T \bar{x}\right). \qquad (17)$$

A matrix containing derivatives of the sigmoid vector is chosen as

$$s'(z) = \begin{bmatrix} 0 & \cdots & 0 \\ \frac{\partial s(z_1)}{\partial z_1} & & 0 \\ & \ddots & \\ 0 & & \frac{\partial s(z_{n_2})}{\partial z_{n_2}} \end{bmatrix}. \qquad (18)$$

In this instance, a SHL NN will be trained online to approximate the model error function

$$\begin{aligned} &\Delta(x, \dot{x}, d_{cmd}) = \\ &f\left(x, \dot{x}, \text{sat}\left(d_{cmd}, \bar{d}\right)\right) - \hat{f}\left(x, \dot{x}, \text{sat}\left(d_{cmd}, \hat{\bar{d}}\right)\right) \end{aligned} \qquad (19)$$

which implies one appropriate choice for the NN inputs (x, $\dot{x}$, and d_{cmd}). The dependency of model error on system signals can be expressed as

$$\begin{aligned} &\Delta(x, \dot{x}, d_{cmd}) = \\ &\Delta\left(x, \dot{x}, \hat{f}^{-1}\left(x, \dot{x}, n\left(f_{rm}(x_{rm}, \dot{x}_{rm}, x_c, \dot{x}_c), e_{rm}, n_{ad}, \|Z\|\right)\right)\right) \end{aligned} \qquad (20)$$

Now, command tracking error is defined as $e_c^T = \begin{bmatrix} (x_c - x)^T & (\dot{x}_c - \dot{x})^T \end{bmatrix}$. Taking advantage of dependencies, one can express

$$\Delta(x, \dot{x}, d_{cmd}) = \Delta\left(x_c, \dot{x}_c, e_c, e_{rm}, n_{ad}, \|Z\|\right) \qquad (21)$$

By the universal approximation theorem for SHL NNs, there exists a set of ideal weights W^* and V^* that brings the output of the NN to within an e -neighborhood of the model error Δ. The matrices W^* and V^* can be defined as the values of W and V that minimize $\bar{e}$, where

$$\sup_{\bar{x} \in D} \left\| \Delta\left(x_c, \dot{x}_c, e_c, e_{rm}, n_{ad}, \|Z\|\right) - W^T s\left(V^T \bar{x}\right) \right\| = \bar{e} \qquad (22)$$

and D is a compact set. This value ($\bar{e}$) can be made arbitrarily small by selection of a sufficient number of middle layer neurons (n_2). The NN input vector can be chosen to be $x_{in}^T = \begin{bmatrix} x_c^T & \dot{x}_c^T & e_c^T & e_{rm}^T & n_{ad}^T & \|Z\| \end{bmatrix}$.

Substitution of Eqn 21 into Eqn 11 reveals the implied assumption that fixed point solution for n_{ad} exists. To guarantee existence and uniqueness of a solution, we can assume that the map $n_{ad \mapsto \Delta}$ is a contraction. It can be shown that this is equivalent to the following two conditions, which imposes restrictions on the choice for $\hat{f}$:

(i) $\quad \text{sgn}(\partial \hat{f} / \partial d) = \text{sgn}(\partial f / d)$

(ii) $\quad \left|\partial \hat{f} / \partial d\right| > \left|\partial f / \partial d\right| / 2 > 0$

2.3. Reference Model Selection

The requirements on the design of n_{crm} necessary for the theorem below are based on first considering the stability properties of an isolated non-adaptive system. The approach is similar to the integrator backstepping method [16], where existence of a Lyapunov function for a part of the overall system is supposed. This permits an analysis of the overall system with the isolating assumption removed. The isolated non-adaptive system represents the plant when reference model tracking error (e_{rm}) is zero, and NN adaptation is complete ($W = W^*$ and $V = V^*$).

The requirements on the design of n_{crm} are the existence of a Lyapunov function for the isolated non-adaptive system

$$\ddot{x} = f\left(x, \dot{x}, \text{sat}\left(\hat{f}^{-1}\left(x, \dot{x}, n_{crm} - n_{ad}^*\right), \bar{d}\right)\right) \qquad (23)$$

where $n_{ad}^* = W^{*T} s\left(V^{*T} \bar{x}\right)$. The reference model can be taken as

$$n_{crm} = \begin{bmatrix} K_{pc} & K_{dc} \end{bmatrix} e_{crm} \qquad (24)$$

where $K_{dc} > 0 \in \Re^{n \times n}$ and $K_{pc} > 0 \in \Re^{n \times n}$ are diagonal matrices containing desired second-order linear command reference model error dynamics, chosen such that $A_c = \begin{bmatrix} 0 & I \\ -K_{pc} & -K_{dc} \end{bmatrix}$ is Hurwitz. The reference model command tracking error is determined by $e_{crm}^T = \begin{bmatrix} (x_c - x_{rm})^T & (\dot{x}_c - \dot{x}_{rm})^T \end{bmatrix}$.

When considering the isolated system design for the non-adaptive subsystem, reference model tracking is zero (so $e_{crm} = e_c$), and one can take

$$L_c(e_c) = \frac{e_c^T P_c e_c}{2} \tag{25}$$

as a candidate Lyapunov function for the subsystem, where $P_c \in \Re^{2n \times 2n}$ is the positive definite solution to the Lyapunov equation $A_c^T P_c + P_c A_c + Q_c = 0$. This Lyapunov function candidate has the property that will be required for the theorem below. Specifically, it satisfies

$$\left\| \frac{\partial L_c}{\partial e_c} \right\| \le b \|e_c\| = l_{\max}(P_c) \|e_c\| . \tag{26}$$

Now, one can write

$$\dot{e}_c = \begin{bmatrix} \dot{x}_c - \dot{x} \\ -f\left(x, \dot{x}, \mathrm{sat}\left(\hat{f}^{-1}\left(x, \dot{x}, n_{crm} - n_{ad}^*\right), \bar{d}\right)\right) \end{bmatrix} \tag{27}$$

When no actuator is beyond a displacement saturation limit (i.e., $d = d_{cmd}$), this becomes

$$\dot{e}_c = \begin{bmatrix} \dot{x}_c - \dot{x} \\ -f\left(x, \dot{x}, \hat{f}^{-1}\left(x, \dot{x}, n_{crm} - n_{ad}^*\right)\right) \end{bmatrix} \tag{28}$$

which can be written as

$$\dot{e}_c = \begin{bmatrix} \dot{x}_c - \dot{x} \\ -n_{crm} - e \end{bmatrix} = A_c e_c + Be \tag{29}$$

where e is the instantaneous residual network approximation error corresponding to the ideal weights, which is subject to $\|e\| < \bar{e}$. So

$$\dot{L}_c \le -\tfrac{1}{2} e_c^T Q_c e_c + \bar{e} \|P_c B\| \|e_c\| \tag{30}$$

and for positive scalars $a = \frac{1}{2} l_{\min}(Q_c)$ and $g = \bar{e}\|P_c B\|$

$$\dot{L}_c \le -a \|e_c\|^2 + g \|e_c\| \tag{31}$$

will be true for a range of allowable commands, plant dynamics (f), and actuator capabilities ($\bar{d}$) (i.e., those that are non-saturating). When one also allows some degree of saturation, the corresponding values for a will decrease and g will increase.

2.4. Adaptation and Guaranteed Boundedness

The following theorem guarantees ultimate boundedness of the reference model states, NN weights, the plant states, and actuator signals.

Theorem 1: *Given the system described in Eqns 1-31, together with the additional assumptions:*

1. *A dynamic inverse of the plant dynamics (f) exists for $x, \dot{x} \in D$ and $d \in \Re^m$.*
2. *A Lyapunov function $L_c(e_c, t)$ for the states of the isolated non-adaptive system, defined for all $e_c \in D$ and $t \in [0, \infty)$ with continuous first-order partial derivatives with respect to e_c and t, and class K_∞ functions g_1 and g_2 satisfies, for a set of allowable plant dynamics (f) and actuator models (g),*

$$g_1(\|e_c\|) \le L_c(e_c, t) \le g_2(\|e_c\|) \tag{32}$$

$$\frac{\partial L_c}{\partial t} + \frac{\partial L_c}{\partial e_c} \dot{e}_c \le -a \|e_c\|^2 + g \|e_c\| \tag{33}$$

$$\left\| \frac{\partial L_c}{\partial e_c} \right\| \le b \|e_c\| \tag{34}$$

3. *External commands are bounded $\left\| \begin{bmatrix} x_c^T & \dot{x}_c^T \end{bmatrix} \right\| \le \bar{x}_c$.*
4. *The norm of the ideal NN weights is bounded by a known value $\|Z^*\| \le \bar{Z}$.*
5. *The adaptive law for the NN weights is given by*

$$\dot{W} = -\left[\left(s - s' V^T \bar{x}\right) r + k\left(\|e_c\| + \|e_{rm}\| + \|e_c\|\|e_{rm}\|\right) W\right] \Gamma_w \tag{35}$$

$$\dot{V} = -\Gamma_v \left[\bar{x} r W^T s' + k\left(\|e_c\| + \|e_{rm}\| + \|e_c\|\|e_{rm}\|\right) V\right] \tag{36}$$

6. *The learning rate matrices (Γ_w and Γ_v) are diagonal and positive definite ($\Gamma_w > 0$, $\Gamma_v > 0$).*
7. *The so-called e-modification parameter (k) in Eqns 35 and 36 is sufficiently high, with lower limit discussed in the proof. Robustifying term gain (K_r) is sufficiently high, with lower limit discussed in the proof. Command tracking error dynamics and reference model tracking error dynamics have sufficient bandwidth, with lower limits discussed in Remark 5 below.*
8. *Initial conditions are within the largest level set of the Lyapunov function utilized in the proof that lies entirely within D.*

All system signals in the closed loop system are ultimately bounded.

Proof: see[17].

Remark 3: The ultimate bound on reference model tracking error can be reduced by increasing $l_{\min}(Q)$.

Remark 4: The ultimate bound on command tracking error can be reduced by increasing a.

Remark 5: The theorem requires that the largest level set of the Lyapunov function utilized in the proof that lies entirely within the set $D_{\tilde{Z}} \times D_{e_{rm}} \times D_{e_c}$, which is the set of allowed NN weight, reference model tracking, and command tracking errors. The set $D_{\tilde{Z}} \times D_{e_{rm}} \times D_{e_c}$ is the largest subspace of these signals that maps into domain D for allowed commands ($\left\| \begin{bmatrix} x_c^T & \dot{x}_c^T \end{bmatrix} \right\| \le \bar{x}_c$) and allowed ideal NN weights ($\|Z^*\| \le \bar{Z}$). Furthermore, this largest level set must contain the compact set outside-of-which this Lyapunov function does not increase, i.e., the smallest invariant set. For a given set $D_{\tilde{Z}} \times D_{e_{rm}} \times D_{e_c}$, this is ensured by increasing $\lambda_{\min}(Q)$ and a (which reduces the size of this compact set in the direction of tracking errors) and adjusting Γ_w and Γ_v (which changes the relative size of the level set in the direction of NN weight errors) to insure that these conditions are satisfied.

3. Domain of Attraction

The largest level set of the Lyapunov function used in the proof that is contained within domain D is an estimate for the domain of attraction of this system (for the equilibrium defined by the command). The Lyapunov function, utilized in the Theorem 1 proof, is

$$L\left(e_c, e_{rm}, \tilde{W}, \tilde{V}\right) = \tfrac{1}{2}\left[L_c(e_c) + e_{rm}^T P e_{rm} + tr\left(\tilde{W}\Gamma_w^{-1}\tilde{W}^T\right) + tr\left(\tilde{V}^T\Gamma_v^{-1}\tilde{V}\right)\right]. \quad (37)$$

Previous results without PCH gave boundedness along trajectories where $n_h = 0$ when n_h is designed as described. Now, boundedness is ensured for trajectories where $|n_h|$ is sufficiently small such that $\dot{L}_c(e_c)$ remains negative semi-definite outside the compact set discussed in the proof, which includes the unsaturated case ($n_h = 0$) as a subset. Therefore, the level set that can be used as an estimate of domain of attraction is *at least as large* as without PCH. However, the level set can be very conservative when the limiting factor on D is Condition #3 of Theorem 1. An example is used to illustrate.

Domain of Attraction Example

Consider the single-degree-of-freedom short-period approximation of the longitudinal motion of an open-loop statically unstable airplane of the form

$$\ddot{x} = f(x, \dot{x}, d) = \hat{f}(x, \dot{x}, d) = M_a x + M_q \dot{x} + M_d d \quad (38)$$

$$d = \hat{d} = \text{sat}(d_{cmd}, 1) \quad (39)$$

where $M_a = 1$, $M_q = -1$, $M_d = -1$, $K_p = K_{pc} = 1$, and $K_d = K_{dc} = 2$. Here, the command, and therefore the equilibrium, is chosen to be the origin. Due to the unstable plant dynamics and the saturating input, this plant has a null-controllable domain (domain in which no control policy can bring the system to the origin) which can be shown to be

$$\dot{x} > d_1 x + d_0 \quad \text{and} \quad \dot{x} < d_1 x - d_0 \quad (40)$$

where

$$d_0 = \frac{2M_d}{M_q + \sqrt{M_q^2 + 4M_a}} \;\; \& \;\; d_1 = \frac{M_q}{2} - \frac{\sqrt{M_q^2 + 4M_a}}{2} \quad (41)$$

One is able to make a phase portrait by assuming the NN adaptation is complete and perfect ($e \approx 0$) or, alternatively, that the approximate dynamic inversion is perfect. In either case, model tracking error (e_{rm}), $\tilde{W}$, and $\tilde{V}$ will remain zero if initially zero and a phase portrait can be drawn for the remaining two system states (e_c). Consider a Lyapunov function for these remaining non-trivial states as $L_c(e_c) = \dfrac{e_c^T P_c e_c}{2}$, which satisfies

$$\begin{aligned}\dot{L}_c &= -\tfrac{1}{2} e_c^T Q_c e_c \\ &+ e_c^T P_c B \begin{bmatrix} -\left(M_a + K_{pc}\right)x - \left(M_q + K_{dc}\right)\dot{x} - \ldots \\ \text{sat}\left(-\left(M_a + K_{pc}\right)x - \left(M_q + K_{dc}\right)\dot{x}, |M_d|\right) \end{bmatrix}\end{aligned} \quad (42)$$

In this case, Condition #3 of Theorem 1 is met in the domain where $\dot{L}_c(e_c) \le -a\|e_c\|^2$ which can be found by minimization of $L_c(e_c)$ subject to $\dot{L}_c(e_c) = 0$, with e_c as the independent variable, excluding the origin. Without PCH, the level set is a minimization of $L_c(e_c)$ subject to

$$\left|-\left(M_a + K_{pc}\right)x - \left(M_q + K_{dc}\right)\dot{x}\right| = |M_d|. \quad (43)$$

These level sets can be found symbolically or numerically as $e_c^T P_c e_c < \approx 0.9533$ with PCH, and $e_c^T P_c e_c < \dfrac{2}{3} \approx 0.6667$ without PCH.

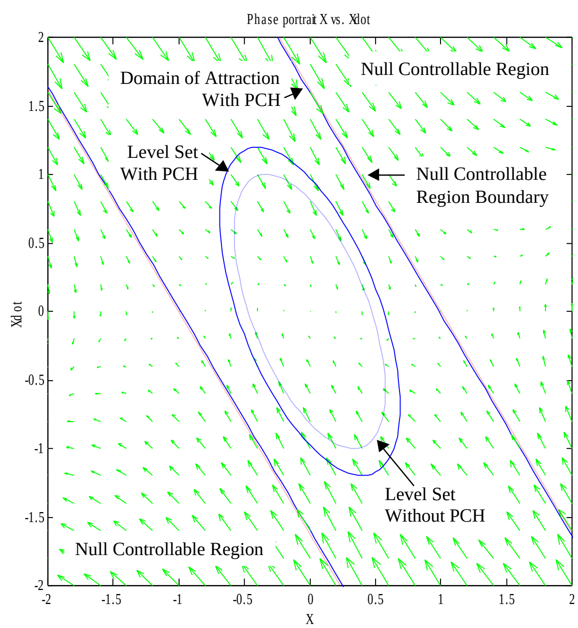

Figure 2 – Phase portrait of input-saturation example assuming ideal dynamic inversion

Figure 2 shows the resulting phase portrait. The null-controllable region and a numerically computed domain of attraction with PCH are labeled. The actual domain of attraction with PCH was determined numerically using the trajectory reversing method [16]. Note that the largest possible domain of attraction is practically achieved by this controller. The guaranteed level sets with and without PCH are also shown.

The actual domain of attraction without PCH cannot be shown on a phase portrait because the trivial solutions for e_{rm}, $\tilde{W}$, and $\tilde{V}$ do not hold.

Now, a higher bandwidth design is explored. In this case desired response has been changed to $K_p = K_{prm} = 4$, $K_d = K_{drm} = 4$ The level sets are found to be $e_c^T P_c e_c < \approx 0.06748$ with PCH, and $e_c^T P_c e_c < \approx 0.03116$ without PCH. The new phase portrait is shown in Figure 3. The achieved domain of attraction with PCH is unchanged by the change in bandwidth. That is, the largest possible domain of attraction is still practically achieved by the controller. The amount of improvement in terms of level sets due to PCH (as a ratio of "before" and "after" PCH applied) is more pronounced.

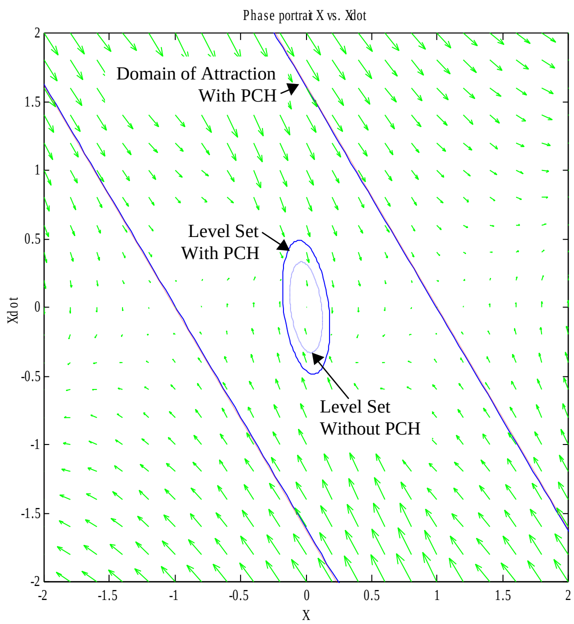

Figure 3 – Phase portrait of input-saturation example - high bandwidth design; actual domain of attraction with PCH is unchanged

4. Conclusions

An adaptive control method has been described that allows input saturation to be removed from the adaptation response. These saturation input characteristic is dealt with as a non-adaptive control synthesis problem. Tracking performance and boundedness of all system signals is guaranteed. The characteristics of the architecture presented, including tolerance to errors in the estimate of actuator position, are good enough that this method should be considered for any application involving an adaptive control component that cancels modeling error in a dynamic inverse.

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References

1 Calise, A. and Rysdyk, R., "Nonlinear Adaptive Flight Control Using Neural Networks," Control Systems Magazine, December 1998.

2 Lewis, F., "Nonlinear Network Structures for Feedback Control," *Asian Journal of Control*. Vol. 1, No. 4, December 1999.

3 Brinker, J. and Wise, K., "Flight Testing of a Reconfigurable Flight Control Law on the X-36 Tailless Fighter Aircraft," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, 2000.

4 Calise, A., Lee, S., and Sharma, M., "Development of a Reconfigurable flight control law for the X-36 tailless fighter aircraft," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, 2000.

5 Sharma, M., Calise, A., and Corban, J., "Application of an Adaptive Autopilot Design to a Family of Guided Munitions," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, 2000.

6 Johnson, E., Calise, A., Rysdyk, R., and El-Shirbiny, H., "Feedback Linearization with Neural Network Augmentation Applied to X-33 Attitude Control," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, 2000.

7 Wang, H. and Sun, J., "Modified Model Reference Adaptive Control with Saturated Inputs," *Proceedings of the 31st Conference on Decision and Control*, IEEE, 1992.

8 Jiang, T., *Combined Model- and Rule-based Controller Synthesis With Application to Helicopter Flight Control*, Ph.D. Thesis, Georgia Institute of Technology, 1998.

9 Leonessa, A., Haddad, W., and Hayakawa, T., "Adaptive Control for Nonlinear Uncertain Systems with Actuator Amplitude and Rate Saturation Constraints," Draft Submitted to *American Controls Conference*, 2001.

10 Monopoli, R., "Adaptive Control for Systems with Hard Saturation," *Proceedings of the IEEE Conference on Decision & Control*, 1975.

11 Ohkawa, F. and Yonezawa, Y., "A Discrete Model Reference Adaptive Control System for a Plant With Input Amplitude Constraints," *International Journal of Control*, Vol. 36. No. 6, 1982.

12 Karason, S. and Annaswamy A., "Adaptive Control in the Presence of Input Constraints," *IEEE Transactions on Automatic Control*, Vol. 39, No. 11, 1994.

13 Annaswamy, A. and Wong, J., "Adaptive Control in the Presence of a Saturation Non-linearity," *International Journal of Adaptive Control and Signal Processing*, Vol. 11, 3-19, 1997.

14 Hanus, R., Kinnaert, M, and Henrotte J.-L., "Conditioning Technique, a General Anti-windup and Bumpless Transfer Method," *Automatica*, Vol. 23, No. 6, 1987.

15 Hornik, K., Stinchombe, M., and White, H., "Multilayer Feedforward Networks are Universal Approximators," *Neural Networks*, Vol. 2, 1989.

16 Khalil, H., *Nonlinear Systems*, 2nd Edition, Prentice Hall, 1996.

17 Johnson, E., *Limited Authority Adaptive Flight Control*, Ph.D. Thesis, Georgia Institute of Technology, 2000.