

This problem is of energy management.

For a given radius  $R$ , the spacecraft speed is

$$v = \sqrt{\frac{\mu}{R}} \text{ and the reduced energy is } E = \frac{-\mu}{2R}$$

The reduced power that goes into the spacecraft total energy is  $P_{in} = \frac{\vec{F} \cdot \vec{v}}{m} = \frac{Fv}{m}$  if the nozzle is oriented right (opposite to the speed vector).

So: 
$$P_{in} = \frac{F}{m} \sqrt{\frac{\mu}{R}}$$

so: 
$$\frac{dE}{dt} = \frac{F}{m} \sqrt{\frac{\mu}{R}} = \frac{F}{m} \sqrt{-2E} \quad \text{since } E = \frac{-\mu}{2R}$$

so 
$$\frac{dE}{dt} = \frac{F}{m} \sqrt{-2E} \quad \text{or} \quad \frac{dE}{\sqrt{-2E}} = \frac{F}{m}$$

integrating: 
$$-\left[ \sqrt{-2E} - \sqrt{-2E_0} \right] = \frac{F}{m} (t - t_0)$$

$t = t_0 \Rightarrow E = \frac{-\mu}{2R_0}$

at  $t_{\text{escape}}$ ,  $E = 0$

So: 
$$\sqrt{\frac{\mu}{R_0}} = \frac{F}{m} (t - t_0)$$

or: 
$$t = t_0 + \frac{m}{F} \sqrt{\frac{\mu}{R_0}}$$