Problem [10pt] Develop the set of differential equations that describes the motion of an aircraft model in a wind tunnel, which is free to roll and yaw about its C.G., but the CG remains fixed.

FYI, the unconstrained lateral-directional set of equations is

$$(1) \quad \left(\frac{d}{dt} - Y_v\right)\Delta v - Y_p\Delta p + (u_0 - Y_r)\Delta r - g\cos\theta_0\Delta\phi = Y_{\delta_r}\Delta\delta_r$$

$$(2) \quad -L_v\Delta v + \left(\frac{d}{dt} - L_p\right)\Delta p - \left(\frac{I_{xz}}{I_x} + L_r\right)\Delta_r = L_{\delta_a}\Delta_a + L_{\delta_r}$$

$$(3) \quad -N_v\Delta v - \left(\frac{I_{xz}}{I_z}\frac{d}{dt} + N_p\right)\Delta + \left(\frac{d}{dt} - N_r\right)\Delta r = N_{\delta_a}\Delta\delta_a + N_{\delta_r}\Delta_r$$

You may want to figure out "what simplifies" if the aircraft CG is not free to move anymore Give some idea about the "modes" of the system and compare them with the unconstrained lateral-directional modes.

Solution: The first equation (1) is the lateral force equation. when the CG is fixed it is not valid anymore. When dealing with Δv, it is easier to think of it as $u_0 \Delta\beta$, like in the book, when the yaw-only equation was derived — then it is even better to think of $\Delta\beta$ as $\Delta\psi$: $\Delta\beta = \Delta\psi$ which is easy to measure in a wind tunnel. Then:

$$(2) \Rightarrow \quad -L_v u_0 \Delta\psi + \left(\frac{d}{dt} - L_p\right)\Delta p - \left(\frac{I_{xz}}{I_x} + L_r\right)\Delta r = L_{\delta_a}\delta_a + L_{\delta_r}\delta_r$$

$$(3) \Rightarrow \quad -N_v u_0 \Delta\psi - \left(\frac{I_{xz}}{I_z}\frac{d}{dt} + N_p\right)\Delta p + \left(\frac{d}{dt} - N_r\right)\Delta r = N_{\delta_a}\Delta\delta_a + N_{\delta_r}\delta_r$$

we need 2 more equations:

$$\frac{d}{dt}\psi = r$$

$$\frac{d}{dt}\phi = p$$

and the deal is closed.

Concerning modes, I have no idea what they are — I would have to rely on an eigenvector analysis