Problem [10pt] In class, we have studied the longitudinal dynamics of an aircraft with fixed elevator. In particular, we have studied the short period of the aircraft for fixed elevator (elevator setting is 0). You are now asked how the short period core characteristics change if the elevator is let loose. After discussing the evolution of the pertinent stability derivatives, give some insight about how the natural frequency and damping change. This is an open subject - Try and formulate an answer which is not too verbose, yet not too "dry" (eg equations only with no explanations). You should have reached conclusions by the end of your write-up.

This is a typical, "ill posed" question - So I need to make a few assumptions before answering it. The main assumption is that the elevator ~~ma~~ is always "at equilibrium". This is not obvious, given that inertia effects could play in, thereby generating the elevator's own "short period"! From the book, page 69, we have, for a free elevator:

$$C'_{m_\alpha} = C_{L_{\alpha_w}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - C'_{L_{\alpha_t}}\eta V_H\left(1 - \frac{d\varepsilon}{d\alpha}\right). \quad (2.62)$$

with $C'_{L_{\alpha_t}} = C_{L_{\alpha_t}}\left(1 - \underbrace{\frac{C_{L_{\delta_e}}}{C_{L_{\alpha_t}}}\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}}}_{\text{this quantity is usually positive (2.56)}}\right)$

So usually $0 < C'_{L_{\alpha_t}} < C_{L_{\alpha_t}}$ (that is, the simplest)

So $C'_{m_\alpha} > C_{m_\alpha}$. The best equation to look at is eq. (4.35): if $C'_{m_\alpha} > C_{m_\alpha}$, $0 < -M'_\alpha \leq -M_\alpha$ the natural frequency goes <u>DOWN</u>

Now for damping:

From equation (3.78), (3.79) $C'_{mq} = -2 C'_{L\alpha_t} \eta V_H \frac{l_t}{\bar{c}}$

since $C'_{L\alpha_t} < C_{L\alpha_t}$, we get

$$0 > C'_{mq} > C_{mq}$$

From (3.92) $C'_{m\dot{\alpha}} = -2 C'_{L\alpha_t} \eta V_H \frac{l_t}{\bar{c}} \frac{d\varepsilon}{d\alpha} < 0$.

So: $0 > C'_{m\dot{\alpha}} > C_{m\dot{\alpha}}$

So in the equation

$$\Delta\ddot{\alpha} - (M'_q + M'_{\dot{\alpha}})\Delta\dot{\alpha} - M'_\alpha \Delta\alpha = 0 \quad (4.35)$$

the term $-(M'_q + M'_{\dot{\alpha}})$ goes down

the damping coefficient ζ is $\frac{-(M'_q + M'_{\dot{\alpha}})}{2\sqrt{-M'_\alpha}}$

may go up or down.

~~So~~ a more verbose thing would have earned you 10 pts too ---

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