Singular Perturbations in Control of Nonlinear Aerospace Systems

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Outline

- Problem Statement
  - Motivation
  - Immediate Applications

- Background: Geometric Singular Perturbation Theory
  - Standard vs. Non-standard Form
  - Two-stage designs

- Current Work & Results
  - Literature review
  - Problems studied

- Future Research Directions
The Future?
Intelligent Organic Aircraft
Why This Is A Hard Problem

10 Independent Morphing DOF

\[
\begin{align*}
\dot{\sigma} &= f(\sigma, \omega) \\
\dot{\omega} &= g(\sigma, \omega) + h(\sigma, \omega, p)u
\end{align*}
\]

Affine in Control

\[
\begin{align*}
\dot{\sigma} &= f(\sigma, \omega) \\
\dot{\omega} &= g(\sigma, \omega, u) + h(\sigma, \omega, p, u)
\end{align*}
\]

Non-Affine in Control
Why This Is A Hard Problem

**Issues:**
- Multiple time-scale behaviour
- Nonlinearity in control
- Real-time/On-board controller
- Time-varying unknown dynamics

**Common Solutions:**
- Gain Scheduling
- Offline, pre-computed control
- Linear system representations
Applications

Aggressive Maneuvering

Non-Minimum Phase

Hover

Approach & Landing

PCA

Tuning with throttles (engines)
Tuning with rudder
Model Description

Stabilization of non-standard nonlinear singularly perturbed systems

\[ \dot{x} = f(x, z, \delta) \]
\[ \epsilon_1 \dot{\delta}_1 = f_{\delta_1}(\delta_1, u_1) \]
\[ \epsilon_2 \dot{z} = g(x, z, \delta, \epsilon_2) \]
\[ \epsilon_3 \dot{\delta}_2 = f_{\delta_2}(\delta_2, u_2) \]

\( x \) is the vector of slow variables,
\( z \) is the vector of fast variables,
\( \epsilon \) captures the time scale property

System Properties:

Unstable open-loop
Unknown time-scale separation (singular perturbation parameters)
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Geometric Singular Perturbation Theory
(Fenichel, 1979)

Derivatives wrt slow time scale $t$
\begin{align*}
\dot{x}_1 &= -x_1 - z \\
\dot{x}_2 &= -x_2 - z \\
\epsilon \dot{z} &= -z
\end{align*}

$\tau = \frac{t - t_0}{\epsilon}$
\begin{align*}
x'_1 &= -\epsilon x_1 - \epsilon z \\
x'_2 &= -\epsilon x_2 - \epsilon z \\
z' &= -z
\end{align*}

Develop reduced-order models. Substitute $\epsilon = 0$
\begin{align*}
z &= 0 \\
\dot{x}_1 &= -x_1 \\
\dot{x}_2 &= -x_2 \\
x'_1 &= 0 \\
x'_2 &= 0 \\
z' &= -z
\end{align*}

Slow Subsystem
Fast Subsystem
Geometric Singular Perturbation Theory

- Reduced-order models approximate the behaviour of the complete system

**Complete System**

\[
\begin{align*}
\dot{x}_1 &= -x_1 - z \\
\dot{x}_2 &= -x_2 - z \\
\epsilon \dot{z} &= -z
\end{align*}
\]

**Slow Subsystem**

\[
\begin{align*}
z &= 0 \\
\dot{x}_1 &= -x_1 \\
\dot{x}_2 &= -x_2
\end{align*}
\]

**Fast Subsystem**

\[
\begin{align*}
x_1 &= 0 \\
x_2 &= 0 \\
\dot{z} &= -z
\end{align*}
\]
Historically: Control Using Geometric Singular Perturbation Methods

In the slow time-scale $t$

\[ \dot{x} = f(t, x, z, u(x, z), \epsilon) \]
\[ \epsilon \dot{z} = g(t, x, z, u(x, z), \epsilon) \]

Set $\epsilon = 0$

\[ \dot{x}_s = f(t, x_s, h(t, x_s), u(x_s), 0) \]
\[ z = h(t, x_s, u(x_s)) \]

Slow Subsystem

In the fast time-scale $\tau = \frac{(t-t_0)}{\epsilon}$

\[ \dot{x} = \epsilon f(t_0 + \epsilon \tau, x, z, u(x, z), \epsilon) \]
\[ \dot{z} = g(t_0 + \epsilon \tau, x, z, u(x, z), \epsilon) \]

Fast Subsystem
Two-stage Design / Composite Control

1. Design Slow Control $u_s(x_s)$ to stabilize/regulate/track the slow subsystem

   $\dot{x}_s = f(t, x_s, h(t, x_s), u(x_s), 0)$

2. Design Fast Control $u_f(x_s, z)$ to ensure the root $h(t, x, u_s(x_s))$ is asymptotically uniformly stable root of fast subsystem

   $\dot{z} = g(t_0, x_s, z, u(x_s, z), 0)$

   $u = u_s + u_f$

Objective: eliminate requirement of solving for $h(t,x,u)$
Problems Studied in Literature

- Two-time scale ‘**Standard** Singular Perturbation Model’
  - Kokotovic (1986), Christofides (1996), …

\[
\begin{align*}
\dot{x} &= f(t, x, z, u(x, z), \epsilon) \\
\epsilon \dot{z} &= g(t, x, z, u(x, z), \epsilon)
\end{align*}
\]

- Assumes algebraic equation has unique real root for the fast states “in some domain”
- Aircraft results in literature assumes coupling between control effectors and slow dynamics is negligible, leads to approximate bounded tracking results

- ‘**Non-Standard** Singular Perturbation Model’
  - No unique real root for the fast states
  - No aircraft results in literature
  - Fridman (2001): Optimal control for regulation of the states
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▪ Current Work & Results
  – Problems studied: F/A-18A, DC-8, Hover control

▪ Future Research Directions
Example 1: Slow State Tracking, Non-Standard

Objective:
Execute turning maneuver at specified angle-of-attack profile and zero sideslip angle

Slow states to be tracked:
- Angle-of-attack
- Sideslip angle
- Heading angle

Model:
- Nonlinear
- 6-DOF
- F/A-18A Hornet in stability axes

Slow states: \( \mathbf{x} = [M, \alpha, \beta, \phi, \theta, \psi]^T \)
Fast states: \( \mathbf{z} = [p, q, r]^T \)
Control variables: \( \mathbf{u} = [\delta_e, \delta_a, \delta_r]^T \)
Example 1 Non-Standard F/A-18A Model

Issues:
- The system is numerically stiff with unknown time-scale separation
- Multiple roots for the fast states (angular rates)

Approach:
- Model-reduction via Geometric Singular Perturbation Theory
- Approximate solution of algebraic equations via centre manifold theorem
- Two-stage design for approximate manifold

Benefits:
- Extends two-stage design procedure to large class of time-scale systems
- Exact knowledge of perturbation parameter not required
- Robustness guaranteed for range of perturbation parameter values
  - Asymptotic stabilization and bounded tracking is guaranteed

Example 1: Slow States
Example 1: Fast States and Controls
Handling Tracking Requirements, Non-Standard

- For the complete system
  \[
  \dot{x} = f(x, z) + g(x, z)u \\
  \epsilon z = l(x, z) + k(x, z)u \\
  y = x
  \]

- Generate reduced order models
  \begin{align*}
  \text{Slow Subsystem} \\
  \dot{x} &= f(x, z) + g(x, z)u \\
  0 &= l(x, z) + k(x, z)u \\
  y &= x
  \end{align*}

  \begin{align*}
  \text{Fast Subsystem} \\
  \dot{x} &= 0 \\
  z &= l(x, z) + k(x, z)u \\
  y &= x
  \end{align*}

- Find the manifold \( \mathcal{M}_0 : z = Z_0(x, u) \) such that it satisfies
  \[
  l(x, z) + k(x, z)u = 0
  \]

- Design tracking controller for the reduced system
- Design controller to stabilize the fast states about the manifold
Centre Manifold

- Linearize the system about \((\epsilon = 0, x, Z_0(x,u))\)

\[
x' = \epsilon (f(x,z) + g(x,z)u)
\]

\[
z' = l(x,z) + k(x,z)u
\]

\[
\epsilon' = 0
\]

- Linearized system:

\[
\Delta x' = F \Delta x + F_1 \Delta z + G \Delta u
\]

\[
\Delta z' = L \Delta z + L_1 \Delta x + K \Delta u
\]

- If all the eigenvalues of \(F\) have zero real parts while \(L\) has negative real parts, then \(M_0\) is the centre manifold.
Centre Manifold Theorem

(Carr, 1981)

- **Conditions** on the system:
  - Origin is the stable equilibrium,
  - System vector fields are sufficiently smooth,
  - System has a centre manifold,

- **Statement:**
  For small initial conditions, then the manifold can be approximated to any degree of accuracy.

- Example:
  \[ x = xz + ax^3 + bz^2 x \]
  \[ \epsilon z = -z + cx^2 + dx^2 z \]
  - Algebraic equation: \[ -z + cx^2 + dx^2 z = 0 \]
  - Approximate manifold \[ \Phi_0 = cx^2 \quad |x| < 1 \]
Mathematical Formulation of the Control Law

- Complete Model:
  \[ \dot{x} = f(x, z) + g(x, z)u \]
  \[ \dot{\epsilon} z = l(x, z) + k(x, z)u \]
  \[ y = x \]

- Tracking error: \( e = x - x_r \)
- Error between fast states and approximate manifold: \( \zeta = z - \Phi_0(e, u) \)
- Transform the equilibrium to origin:
  \[ \dot{e} = f^*(e, \zeta + \Phi_0) + g^*(e, \zeta + \Phi_0)u; \]
  \[ \epsilon \dot{\zeta} = l^*(e, \zeta + \Phi_0) + k^*(e, \zeta + \Phi_0)u - \epsilon \Phi_0 \]
Composite Control Design

\[
\dot{e} = f^*(e, \zeta + \Phi_0) + g^*(e, \zeta + \Phi_0)u;
\]

\[
\epsilon \dot{\zeta} = l^*(e, \zeta + \Phi_0) + k^*(e, \zeta + \Phi_0)u - \epsilon \Phi_0
\]

- **Slow Subsystem**
  \[
  \dot{e} = f^*(e, \Phi_0(e, u_s)) + g^*(e, \Phi_0(e, u_s))u_s
  \]

- **Control Law**
  \[
  u_s = -(\tilde{g}(e))^{-1}(K_e e + \tilde{f}(e))
  \]

- **Fast Subsystem**
  \[
  \dot{e} = 0;
  \]

  \[
  \zeta' = l^*(e, \zeta + \Phi_0) + k^*(e, \zeta + \Phi_0)(u_s + u_f)
  \]

- **Control Law**
  \[
  u_f = -(\tilde{k}(e))^{-1}(K_\zeta \zeta + \tilde{I}(e))
  \]

Control applied to the complete system (independent of \(\epsilon\))

\[
\mathbf{u} = u_s(e) + u_f(e, \zeta)
\]

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Lyapunov Analysis

- **Lyapunov Function Candidate:**
  \[
  \nu(e, \zeta) = e^T e + \zeta^T \zeta
  \]

- **Time derivative about closed-loop dynamics:**
  \[
  \dot{\nu} = - \left[ \|e\| \right]^T \left[ \begin{array}{cc} 2K_e & -\beta_1 \\ -\beta_1 & 2\frac{\|e\|}{K_\zeta} \end{array} \right] \left[ \|e\| \right] - \|\zeta\| \|2\Phi_0\|
  \]

- **For** \( \epsilon < \frac{8K_eK_\zeta}{\beta_1^2} \) **stability of the system is guaranteed.**

- **Note:** If the exact solution of manifold is computed, asymptotic stability can be proved.
Example 2: Slow and Fast State Tracking, Non-Standard

**Objective:**
Execute aggressive vertical climb with maximum pitch rate of $25\text{deg/sec}$ followed by roll at a rate of $50\text{deg/sec}$ with sideslip angle constrained to zero.

**Slow states to be tracked:**
- Sideslip angle

**Fast states to be tracked:**
- Roll, pitch, yaw body-axis rates

**Model:**
- Nonlinear
- 6-DOF
- F/A-18A Hornet in stability axes

Slow states: $x = [M, \alpha, \beta, \phi, \theta, \psi]^T$

Fast states: $z = [p, q, r]^T$

Control variables: $u = [\delta_e, \delta_a, \delta_r]^T$
Example 2 Nonlinear Singularly-Perturbed F-18 Model

Issues:
- Unknown time-scale separation
- Fast states restricted to lie on a manifold which is unknown!
- Global results valid only when manifold is unique

Approach:
- Model-reduction via Geometric Singular Perturbation Theory
  - Coordinate transformation
  - Force slow states to track desired reference
  - Enforce manifold to be exactly the fast state reference
- Control using two-stage design

Benefits:
- No assumptions on where nonlinearity appears
- Global asymptotic tracking
- No knowledge of the singular perturbation parameter required

Example 2 Trajectory
Example 2: Fast States and Controls
Example 2: Slow States
Mathematical Formulation of the Control Law

- Complete Model:
  \[
  \dot{x} = f(x, z) + g(x, z)u \\
  \varepsilon \dot{z} = l(x, z) + k(x, z)u \\
  y = \begin{bmatrix} x \\ z \end{bmatrix}
  \]

- Tracking errors:
  \[
  e(t) = x(t) - x_r(t) \\
  \xi(t) = z(t) - z_r(t)
  \]

- Transform the equilibrium to origin:
  \[
  \dot{e} = f(x, z) + g(x, z)u - \dot{x}_r \overset{\Delta}{=} F(e, \xi, x_r, z_r, \dot{x}_r) + G(e, \xi, x_r, z_r)u \\
  \varepsilon \dot{\xi} = l(x, z) + k(x, z)u - \varepsilon \dot{z}_r \overset{\Delta}{=} L(e, \xi, x_r, z_r, \varepsilon \dot{z}_r) + K(e, \xi, x_r, z_r)u
  \]
Step 1. Obtain Reduced-Order Models

\[ \begin{align*}
\dot{e} &= F(e, \xi, x_r, z_r, \dot{x}_r) + G(e, \xi, x_r, z_r)u \\
\dot{\xi} &= L(e, \xi, x_r, z_r, \dot{z}_r) + K(e, \xi, x_r, z_r)u
\end{align*} \]

- **Reduced Slow Subsystem**

\[ \begin{align*}
\dot{e} &= F(e, \xi, x_r, z_r, \dot{x}_r) + G(e, \xi, x_r, z_r)u_0 \\
0 &= L(e, \xi, x_r, z_r, 0) + K(e, \xi, x_r, z_r)u_0
\end{align*} \]

- **Reduced Fast Subsystem**

\[ \begin{align*}
e' &= 0 \\
\dot{\xi}' &= L(e, \xi, x_r, z_r, z_r') \\
&\quad + K(e, \xi, x_r, z_r)(u_0 + u_f)
\end{align*} \]
Step 2. Design Controller for Slow Subsystem

- Force origin as the equilibrium of the closed-loop slow subsystem

\[
\begin{bmatrix}
G(e, \xi, x_r, z_r) \\
K(e, \xi, x_r, z_r)
\end{bmatrix} u_0 = -\begin{bmatrix}
F(e, \xi, x_r, z_r, \dot{x}_r) \\
L(e, \xi, x_r, z_r, 0)
\end{bmatrix} + \begin{bmatrix}
A_e e \\
A_\xi \xi
\end{bmatrix}
\]

\[
\dot{e} = F(e, \xi, x_r, z_r, \dot{x}_r) + G(e, \xi, x_r, z_r)u_0 \\
0 = L(e, \xi, x_r, z_r, 0) + K(e, \xi, x_r, z_r)u_0
\]

\[
\dot{e} = A_e e \\
0 = A_\xi \xi
\]

Resulting Closed-Loop Slow Subsystem
Step 3. Design Controller for Fast Subsystem

To make sure the manifold is stable for all time,

\[
\begin{bmatrix}
G(e, \xi, x_r, z_r) \\
K(e, \xi, x_r, z_r)
\end{bmatrix} u_f = \begin{bmatrix}
0 \\
L(e, \xi, x_r, z_r, 0) - L(e, \xi, x_r, z_r, z_r')
\end{bmatrix}
\]

\[
e' = 0
\]

\[
\xi' = L(e, \xi, x_r, z_r, z_r') + K(e, \xi, x_r, z_r)(u_0 + u_f)
\]

\[
e' = 0
\]

\[
\xi' = A_\xi \xi
\]

Resulting Closed-Loop Fast Subsystem
Step 4. Composite Control Design

- Composite Control: \( u = u_0 + u_f \)

- Or,
  \[
  \begin{bmatrix}
  G(e, \xi, x_r, z_r)
  \\
  K(e, \xi, x_r, z_r)
  \end{bmatrix} u = -\begin{bmatrix}
  F(e, \xi, x_r, z_r, \dot{x}_r)
  \\
  L(e, \xi, x_r, z_r, \dot{z}_r)
  \end{bmatrix} + \begin{bmatrix}
  A_e\xi
  \\
  A_\xi\xi
  \end{bmatrix}
  \]

Resulting Closed-Loop System

\[
\begin{align*}
\dot{e} &= F(e, \xi, x_r, z_r, \dot{x}_r) + G(e, \xi, x_r, z_r)u \\
\epsilon \dot{\xi} &= L(e, \xi, x_r, z_r, \epsilon \dot{z}_r) + K(e, \xi, x_r, z_r)u
\end{align*}
\]

\[
\begin{align*}
\dot{\epsilon} &= A_e \epsilon \\
\epsilon \dot{\xi} &= A_\xi \xi
\end{align*}
\]
Lyapunov Analysis

- **Lyapunov Function Candidate:**
  \[ \nu(e, \zeta) = (1 - d)e^T e + d\zeta^T \zeta, \quad d > 0 \]

- **Time derivative about closed-loop dynamics:**
  \[ \dot{\nu} \leq -\left[ \begin{array}{c} e \\ \xi \end{array} \right]^T \begin{bmatrix} (1 - d)(\alpha_1 - 2\alpha) & 0 \\ 0 & \frac{d}{\varepsilon} \alpha_2 - 2\alpha d \end{bmatrix} \left[ \begin{array}{c} e \\ \xi \end{array} \right] - 2\alpha \nu \]

  \[ \alpha_1 = 2\lambda_{\min}(A_e), \quad \alpha_2 = 2\lambda_{\min}(A_\xi), \quad \alpha > 0 \]

- **If,** \( \epsilon < \frac{\alpha_2}{2\alpha} \), \( \dot{\nu} \leq -2\alpha \nu \)

- **Or,** \( \dot{\nu} \leq -2\alpha \gamma_1 \left\| \begin{array}{c} e \\ \xi \end{array} \right\|^2 \)

- **Thus, global exponential stability can be concluded.**

- **Or,** \( x(t) \rightarrow x_r(t), z(t) \rightarrow z_r(t), t \rightarrow \infty \)
Advanced Control of Aerospace Systems

Issues:
- Non-Minimum Phase
- Various actuator speeds
- Under-actuated

Approach:
Re-pose the problem as slow-state tracking for a singularly perturbed system and exploit inherent multiple-time scale behaviour. Compute the internal state online (real-time) and then uses state-feedback.

Accomplishments:
- No approximations made to render system only slightly non-minimum phase
- Real-Time exact tracking guaranteed by state-feedback control
- Causal: Does not depend and require any prior knowledge of reference

Non-Minimum Phase Control Solution

- Retains force/moment coupling to solve for internal states
- Nothing pre-computed: all online
- Independent of reference

Non-Minimum Phase Control Solution

The natural time-scale decomposition of a vehicle is exploited. The procedure comprises of the following steps:

1. Control the outputs (also the slow states of the system) through the slow actuators (throttle) and the inherent fast states using Lyapunov methods.

2. Design the fast controllers (actuation surfaces) to ensure the fast states follow the desired trajectory designed in Step 1.

3. Use Lyapunov methods to determine analytical bounds on system parameters for robustness and stability.

**NOTE:** Uses coupling between forces and moments that make the system non-minimum phase for computing an online solution for the internal states (the fast states).

Example 3: Non-Minimum Phase, Non-Standard

Objective:
Execute climbing maneuver at constant forward velocity

Slow States to be tracked:
• Body-axis velocities $u, w$

Fast(er) states (internal) to be tracked:
• Pitch attitude $\theta$, body-axis pitch rate $q$

Model:
• Scaled three-dimensional
• DC-8 Benchmark Model
• Sastry, IEEE CDC, 95

Slow states: $x = [x, z, u, w]^T$
Fast states: $z = [\theta, q]^T$
Control variables: $u = [T, M]^T$
Example 3: Non-Minimum Phase, Non-Standard

**Mathematical Model:**

\[
\begin{align*}
\dot{x} &= u \\
\dot{u} &= \cos \theta (u_1 - D \cos \alpha + L \sin \alpha) - \sin \theta (-F_z + D \sin \alpha + L \cos \alpha) \\
\dot{z} &= w \\
\dot{w} &= \sin \theta (u_1 - D \cos \alpha + L \sin \alpha) + \cos \theta (-F_z + D \sin \alpha + L \cos \alpha) \\
\dot{\theta} &= q \\
\dot{q} &= \frac{u_2}{J}
\end{align*}
\]

**Force Relations:**

\[
\begin{align*}
\alpha &= \theta - \tan^{-1} \frac{w}{u} \\
F_z &= \frac{0.3mg}{J} u_2
\end{align*}
\]

**Control Variables:**

- \(u_1\): slow thrust control
- \(u_2\): fast pitch moment
Example 3: Non-Minimum Phase, Non-Standard Conventional Takeoff and Landing

Two-Dimensional Trajectory
Example 3: Non-Minimum Phase, Non-Standard Conventional Takeoff and Landing

Vertical Velocity

Applied Pitching Moment
Example 3: Non-Minimum Phase, Non-Standard Conventional Takeoff and Landing – Internal States

Pitch-Attitude Angle

Pitch Rate
Example 4: Non-Minimum Phase, Non-Standard Helicopter Control

Objective:
Hover control of Autonomous Helicopter. Stabilize the vehicle positions.

Slow-state tracking: $x$, $z$

Model: (Source: Sastry 98')
Nonlinear, three degree-of-freedom unmanned autonomous helicopter

Slow states: $x = [x, z, u, w]^T$
Fast states: $z = [\theta, q]^T$
Control Variables: $u = [T_m, a_1]^T$
Example 4: Non-Minimum Phase, Non-Standard Helicopter Control

Mathematical Model: (Source: Sastry 98')

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
u \\
w
\end{bmatrix}
\]

\[
\begin{bmatrix}
m\dot{u} \\
m\dot{w}
\end{bmatrix} =
\begin{bmatrix}
-qw + F_x \\
qu + F_z
\end{bmatrix} +
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
0 \\
mg
\end{bmatrix}
\]

\[\dot{\theta} = q\]

\[I_y \dot{q} = M\]

Force Model: (Source: Sastry 98')

\[F_x = -T_M \sin a_{1s}\]

\[F_z = -T_M \cos a_{1s}\]

\[M = M_a a_{1s} - F_x h_M + F_z l_M - Q_T\]

Nature of Control Variables:

\(T_M\): slow thrust control

\(a_{1s}\): fast rotor tilt control
Helicopter Open-Loop

**Key Issues:**

Non-minimum phase due to strong coupling between forces & moments

Neutrally stable zero dynamics

Siddarth Anshu & Valasek, John “Control of Non-Standard Singly Perturbed Systems”, in review Automatica
Example 4: Non-Minimum Phase, Non-Standard Helicopter Control – Hover Results

Position Time Histories

Velocity Time Histories

- Forward Position $x(t)$
- Vertical Position $z(t)$
- Forward Velocity $u(t)$
- Vertical Velocity $w(t)$
Example 4: Non-Minimum Phase, Non-Standard Helicopter Control – Internal States

Pitch-Attitude Angle

Pitch-rate
Example 4: Non-Minimum Phase, Non-Standard Helicopter Control – Controls

Main Rotor Thrust

Longitudinal Tilt Angle
Future Research Directions

Multiple-Time Scale Non-Standard Aerospace Systems Control

Known System Dynamics
- Output Feedback
  - State Feedback
    - Slow State Tracking
    - Non-Minimum Phase
    - Simultaneous Slow & Fast Tracking

Unknown System Dynamics
- Output Feedback
  - State Feedback
  - Propulsion - Controlled Aircraft
Future Research Challenges

- Pure output feedback stabilization of multiple time scale aircraft systems

- Control of unknown time-varying multiple-time scale non-standard dynamics

- Non-affine system control:
  - Static feedback techniques available only for stable systems
  - Dynamic feedback designed for systems with monotonic control influence terms
  - Output feedback still remains an open problem
Questions?