

AE3051 Experimental Fluid Dynamics

DIGITAL SAMPLING OF TIME-DEPENDENT SIGNALS

Objective

The primary objective of this experiment is to familiarize the student with digital data acquisition of time-varying signals. This lab covers concepts in frequency analysis of time-varying signals and sampling theory. It also provides an introduction to computer-based data acquisition systems. In this experiment, you will use a computer data acquisition system to sample signals produced by waveforms stored on a compact disk (CD) and converted to an analog electrical signal by a CD player. You will explore issues in sampling, including the Nyquist limit and aliasing, and the use of analog filters. This will help prepare you for future experiments in this laboratory course that employ computerized data acquisition and involve frequency based interpretation of measured data.

Background

Most experimental measurements involve the dimension of **time**. Experimental data is acquired over the course of some time, and during this time the signal can change. In some cases, the actual physical parameter of interest (the *measurand*) may change with time. For example, the velocity in a wind tunnel generally varies with time due to turbulence or variations in the speed of the fan blades used to drive the tunnel. Even when the measurand is nominally constant in time, other parameters that influence the measurement may vary, for example drifts in the measurement device. Thus, the experimenter is often interested in measuring a variable that could be described by the general function (or *waveform*),

$$v=v(t). \quad (1)$$

a) Waveforms, Frequency Content and Discrete Sampling

Fourier Series

One of the simplest time-dependent functions we encounter is the sine (or cosine^{*}),

$$v(t) = A\sin(\omega t + \phi) = A\sin(2\pi nft + \phi) \quad (2)$$

^{*}Either function is acceptable, since $\sin(\omega t) = \cos(\omega t - \pi/2)$, i.e., the two functions are identical except for a phase difference of $\pi/2$ or 90° , meaning that shifted by one-fourth of a cycle, cosine looks just like sine.

where A is the amplitude, ω is the circular frequency (e.g., rad/s), f is the cyclic frequency (e.g., cycles/s, Hertz or s^{-1}), and ϕ is the phase, which represents the time-shift of the sine-wave from some reference time that defines $t=0$. Such a function is often denoted as a *simple harmonic waveform*.

More general periodic waveforms, which repeat themselves with a period T and thus have a frequency $f=1/T$, can be **written as a linear combination of simple harmonic modes**. There is the basic, *fundamental* mode (with frequency f), and *harmonics* of the fundamental mode, with integer multiples of its frequency ($2f$, $3f$, ...). For example, we could describe the vibrations of a tuning fork or the acoustic oscillations in a pipe this way. Mathematically, this linear combination of modes is expressed as a **Fourier series expansion**,

$$v(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(2\pi nft) + b_n \sin(2\pi nft)] \quad (3)$$

where nf represents the frequency of the n^{th} mode ($n=1$ for the fundamental, $n=2$ for the first harmonic, etc.), a_0 represents the steady component of the waveform, and the a_n , b_n are the harmonic coefficients (or amplitudes) of each mode. The steady amplitude, a_0 , is often called the DC component of the waveform, in reference to classical electrical power systems, which are either **D**irect **C**urrent (steady) or **A**lternating **C**urrent (sinusoidal with a zero average).

For example, Figure 1 shows a simple waveform composed of two frequencies, a fundamental mode at 50 Hz and its 9th harmonic (at 500 Hz). Thus the complete waveform is repeated every 20 ms ($=1/\text{fundamental frequency} = 1/50$ s). The waveform shown in the figure also has a DC component. In other words, the signal has a nonzero value when averaged over its period. In general, we can write the DC amplitude as

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} v(t) dt = f \int_{-T/2}^{T/2} v(t) dt. \quad (4)$$

The other coefficients of the Fourier expansion are given by

$$\begin{aligned} a_n &= 2f \int_{-T/2}^{T/2} v(t) \cos(2\pi nft) dt \\ b_n &= 2f \int_{-T/2}^{T/2} v(t) \sin(2\pi nft) dt \end{aligned}, \quad (5)$$

and they can be combined into a complex number,*

$$a_n - i b_n = 2f \int_{-T/2}^{T/2} v(t) e^{-i2\pi nft} dt \quad . \quad (6)$$

The power, P , contained in single mode is given by the square of the amplitude

$$P(n) = a_n^2 + b_n^2 \quad (7)$$

and the phase ϕ (or phase angle) of a mode is given by

$$\phi(n) = \tan^{-1} \left(\frac{b_n}{a_n} \right). \quad (8)$$

A second example that shows the ability of a combination of sine waves to create an arbitrary periodic function is shown in Fig. 2. Five sine waves and a DC component (see Fig. 3) were combined to create a function approaching a square wave. While the constructed function resembles a square wave, it is clear that more sine waves would be needed to produce a sharp square wave.

Fourier Transforms

The procedure outlined above for periodic functions can be extended to general functions, which are not necessarily periodic, by considering any arbitrary function to be periodic with an infinitely long period. This approach leads to the **Fourier Transform**. Given a function $v(t)$, its Fourier Transform $V(f)$ is a complex function defined by

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-i2\pi ft} dt \quad (9)$$

in parallel to the complex Fourier function of equation (6). The function $V(f)$ represents the *information* given by $v(t)$ **transformed from the time domain to the frequency domain**. The transformation is nearly identical in the reverse direction, with simply a change in the phase (note the sign of the exponent), i.e.,

$$v(t) = \int_{-\infty}^{\infty} V(f) e^{+i2\pi ft} df \quad . \quad (10)$$

*Since, $e^{-ix} = \cos x - i \sin x$.

For example, Figure 4 graphically shows the Fourier transforms of various functions, including sine and cosine waves, a rectangle function (Π), a triangle function (Λ) and a constant, or DC, function. The sine, cosine and DC waveforms result in Fourier transforms that are nonzero at a single frequency^{*}; in other words, they contain information at only one frequency (the DC function, which does not change in time, is associated with a frequency of zero). The Fourier transforms of the rectangle and triangle functions result in sinc and sinc² functions, where $\text{sinc}(f) \equiv \sin(\pi f)/\pi f$, which contains information at many frequencies, but with multiple frequency “peaks”.

Instead of looking at the Fourier transform, we often are interested in the *power spectrum* (or *power spectral density, PSD*) of a waveform. This represents the amount of power or energy in a region between f and $f+df$. For real (noncomplex) functions $v(t)$, this is given by

$$PSD(f) = |V(f)|^2 \quad (11)$$

where it is sufficient to consider only $0 < f < \infty$ since the PSD of a real function is symmetric about $f=0$.^{*} Thus the PSD of the rectangle function, $\Pi(x)$ as shown in Figure 4, is the square of its Fourier transform, or sinc²(f) (also shown in Figure 4).

Extensions of the Fourier Transform method have been developed for non-continuous functions, specifically for signals that have been discretely sampled by a computer, data acquisition system, or produced by digital means. These are generally known as Discrete Fourier Transforms. In addition, methods to quickly compute the Fourier Transform have also been developed, e.g., the Fast Fourier Transform. These concepts are described in detail in references 2 and 4. The computer data acquisition system you will use employs these techniques to compute the power and phase spectra of the signals that are sampled in this lab.

Discrete Sampling

In most situations, especially for computer-based data acquisition, the continuous function $v(t)$ is sampled (i.e., the data is acquired) at evenly spaced, discrete intervals in time, separated by an amount Δt . The sampling frequency (or data acquisition rate) is thus $f_s = 1/\Delta t$.

^{*}The negative frequencies relate to phase information for the sine and cosine and do not actually represent different frequencies, i.e., for real functions $v(t)$, it can be shown that $|V(f)| = |V(-f)|$. That means that if you take the absolute value of V , the part of V below 0 frequency looks like a reflection of the part for $f > 0$.

For a given sampling rate, we might ask how accurately the discretely acquired data can reproduce the actual waveform being sampled. The answer depends on the frequency content of the waveform and a special frequency, called the *Nyquist frequency* (f_N), which is half the sampling frequency, i.e., $f_N=f_s/2$. If the waveform contains no components above the Nyquist frequency, then the waveform can be completely determined by the sampled data (assuming no errors in the measurement).** This is known as the *Sampling Theorem*.

As a simple example, consider a single sine wave. If we know we are dealing with a single frequency sine wave, it takes at least two measurements per period to determine its frequency, which means we must sample at twice the sine wave's frequency. If we sample any slower, we actually infer a lower frequency than the actual frequency of the sine wave (you will see this in the lab). This process, by which information at a higher frequency shows up at a lower frequency is known as *aliasing*.

Aliasing occurs for any sampled waveform having components with frequencies above the sampling system's Nyquist frequency, i.e., ($f>f_N$). One way to remove this problem is to filter the data before it is sampled. This can be accomplished by a low pass filter, a filter that only passes frequencies below some cut-off frequency. One would set the cut-off at or below the Nyquist frequency. The high frequency information is thus removed before it can be aliased. In essence, the filter produces a bandwidth limited waveform.

b) Computer Data Acquisition System

Data will be acquired with an board in a computer, utilizing a LabView™ interface. The computer data acquisition board, which measures the voltage of the input signal, essentially consists of a multiplexer, a sample-and-hold device, an analog-to-digital converter, a memory buffer, an interface to the computer's memory, a master clock, and controller (see Figure 5).

The *multiplexer* (MUX) is a switch that connects one of a number of input channels (usually numbered starting at 0) to the *sample-and-hold* (S/H). The input voltage on the channel switched by the MUX “charges up” the sample-and-hold during some time interval, which is a fraction of the sampling period (the time between samples). This circuit is then

A waveform that has information in only a limited range of frequencies is called **bandwidth limited. Due to phase ambiguity, the sampling frequency should actually be more than twice the maximum frequency in the waveform. For example, a sine wave sampled at $0, \pi, 2\pi$, etc. would always have a 0 result and could be confused with a null function.

disconnected from the input voltage, and some of the stored charge is drained from it. From this charge, the original voltage value connected to the S/H is determined, and the result is converted to a digital value by the *analog-to-digital converter* (ADC). The digital value (sometimes referred to as a “word” of data depends on the input voltage, the *voltage range* of the ADC (the *minimum* and *maximum* voltages it reads, e.g., 0-5 V), and its digital dynamic range (number of “bits” = N). The relation between the digitizer output and the voltage input is given by

$$output = \frac{input - minimum}{maximum - minimum} \times (2^N - 1) \quad (12)$$

where *output* has to be an **integer value**. For example, consider a 2 V input into an ADC with a 0-10 V range, and an 8-bit digitizer (2^8 possible values, or digital values of 0-255). The output of the ADC would be a digital value of 51. The digital result is then moved to the buffer memory on the board, and shifted to the computer memory, usually through the computer’s *direct memory access* (DMA) system.

Multiple signal inputs are recorded by using the MUX to cycle through each of the input channels at a rate that must be faster than the overall sampling rate (how often a given channel is read) times the number of input channels being read. In the sequential sampling system illustrated in Fig. 5 (and which is representative of the system you will be using), note that the channels are *not read at exactly the same time*. There is a time delay (**skew**) between when one channel and the next is read. The skew determined by the maximum switching and reading rates of the MUX, S/H and ADC. This is illustrated in Fig. 6. Simultaneous data acquisition systems, which have negligible skew, typically employ multiple, synchronized S/H systems just upstream of the MUX (see Fig. 7).

In this lab, you control the data acquisition process through a software interface called a LabView *virtual instrument* (VI). The VI creates a display on the computer screen that lets you think of the data acquisition system as a box with “knobs”, “dials”, and other displays. For this experiment, the VI allows you to control parameters such as the minimum and maximum voltages read by the acquisition board, the sampling rate (f_s), and the number of samples recorded.

You will also use an analog, electronic filter manufactured by Krohn-Hite. It actually contains two filters, which separately can be switched to be either low pass or high pass filters.

The cutoff-frequency of each is also adjustable, using a combination of a dial and multiplier setting. You will be examining the effects of the filters on time-dependent signals.

Procedure

1. Place the CD containing the waveforms in the CD player and connect the output to the oscilloscope. Set the output (volume) level on the CD player somewhere between its middle and high range. Then use the (software) CD player controls to play each track.
2. Each track contains a different signal. Each signal is periodic, meaning it repeats itself over and over. The waveforms you will see include: **single sine waves** (with different frequencies on different tracks); **sums of sine waves** (e.g., $b_1\sin(2\pi f_1t) + b_2\sin(2\pi f_2t)$), **products of sine waves**; and periodic waveforms that are not sine waves, for example **square waves**, **triangle waves**, or **ramps**. View the output of each waveform on the oscilloscope and try to *quickly* (if you want to leave the lab on time) identify each waveform and what frequency(s) you think it contains. For use in the next step, note the approximate voltage range of one of the waveforms (all the tracks should have about the same maximum and minimum voltages). If you have time later in the lab, you might wish to try listening to each waveform.
3. Connect the CD output to the data acquisition system's inputs (analog input, channel 0), use the VI to set the sampling rate to 22 kHz (22,000 sample/sec), the number of samples recorded to 6000, and the voltage range of the ADC (maximum and minimum voltage limits) to best match the voltage range of the signals you saw on the oscilloscope. Observe the power spectrums of the waveforms on each track of the CD. Take some very brief notes that describe what you see (for example if you see "peaks" and how many, and the frequency associated with a peak). The goal is to determine what kind of signal is on each track of the CD.
4. Find the track containing the simple harmonic waveform (a single sine wave) at ~1 kHz. Acquire data at a 10 kHz sampling rate and for a *few* record lengths, ranging from 4 samples to 5000 samples. (Note, you will have to use the **hold button** on the VI to stop the data acquisition process if you want to change the acquisition parameters.) Observe the power spectra and the time traces of the sampled signal, and note how they change with the choice of record length. Be sure to record how the number of distinct frequencies the Fourier Transform can report varies with the number of samples recorded. The goal here is to examine the frequency resolution of your measurement. Please be careful about picking

too many samples – think about how long it will take to record them (*recording time = number of samples ÷ sampling rate*).

5. For the same ~1 kHz waveform, acquire power spectra at 10 sampling rates. Choose rates above, near and below the frequency of the signal. Choose the number of samples recorded (which you can change for each sampling frequency if you wish) based on the results of step 4. Observe the change in the recorded power spectra and time traces for each sampling frequency. Specifically, record how the frequency of the peak in the power spectrum changes with sampling rate, i.e., how it is **aliased**.
6. Connect the output of the CD player to the input of one of the filters and, in parallel (using a T-connector) to the first input channel (channel 0) of the computer data acquisition system. Connect the output of the filter to the second input channel (channel 1). Use the CD player controls to pick the track containing the sum of three sine waves at three frequencies. Use the VI to set the sampling rate to 18 kHz (per channel) and to record both channels. Set the filter to be a lowpass filter, and vary the filter cutoff frequency between 1 kHz and 10 kHz, and compare the power spectra from the filtered and unfiltered signals. The goal is to find what frequency the filter should be set at to remove the aliasing problems.
7. Set the sampling rate to 22 kHz and the filter cutoff frequency to 10 kHz. Observe the phase plots (phase versus frequency) for the filtered and unfiltered signals. Look only at the phases for the frequencies that showed peaks in the power spectrum. Compare the phases at these frequencies for the filtered and unfiltered signals to see if there are any differences. At each of these frequencies, if the phases are different, record the difference.
8. Use the CD player to play the track with the rapidly sweeping frequency that repeats itself a number of times). Set the sampling rate to 6 kHz and the number of samples (per channel) to 3,000. With the cutoff frequency of the filter set to 800 Hz, record (and save on the computer) power spectra for both the filtered and unfiltered channels. Acquire spectra averaged over 10 measurements and save these too. **Make sure you are displaying the power spectra with a linear scale rather than a dB (log) scale before saving the data.**
9. Remove the output of the lowpass filter from channel 1 on the computer and instead connect the filter output to the input of the second filter, and set the second filter to be a highpass filter. Then connect the output of the (highpass) filter to channel 1 of the computer data acquisition system. Change the track to play the waveform that contains significant amounts of noise on top of a single frequency sine wave. Vary the cutoff frequencies of the two filters and observe the effect on the data acquired.

Data to be Taken

1. A **short** description/identification of each waveform on the CD (one waveform per track).
2. The number of data points (frequencies) in the Fourier Transform-based power spectrum as a function of number of samples recorded.
3. For the simple harmonic waveform, record which frequency has the peak power for each of the (at least) 10 sampling frequencies
4. For the waveform with three frequencies (and the sampling frequency of 18 kHz), determine the maximum filter frequency that is able to produce a faithful record of the frequency content of the waveform *without aliasing*.
5. For the waveform with three fixed frequencies (sampled at 22 kHz and the filter set to 10 kHz), record the phase at the frequencies associated with the power spectrum peaks for both the filtered and unfiltered channels.
6. For the waveform with the rapidly sweeping frequency, record, simultaneously, power spectra for both the filtered and unfiltered signals. **You should acquire a single spectra, and the average of 10 spectra.**

(Only this last item requires you to store data on the computer. However if you wish, you may store results, e.g., spectra, for other measurements you were required to take.)

Data Reduction

1. You will need to calculate the ratio of the power spectra for the filtered data (i.e., the output from the filter) and unfiltered data (i.e., the input to the filter) recorded in step 6 of the Data to be Taken section. Thus you can find the Transfer Function of the filter (output/input at each frequency).

Results Needed for Report

1. A table that identifies the contents of each track of the CD.
2. An expression that relates the number of samples recorded and the sampling frequency to the frequency resolution (the frequency spacing between consecutive points) of the Fourier-Transform based power spectrum.
3. A plot of the frequency peak in the power spectrum as a function of sampling frequency for the simple harmonic waveform.

4. The maximum filter frequency required to prevent aliasing of the data for the waveform with three fixed frequency modes.
5. Plots of the single and average power spectra, filtered and unfiltered, for the waveform with the rapidly sweeping frequency.
6. A plot of the ratio described in step 1 of the data reduction, based on the average results.

Further Reading

1. R. V. Churchill and J. W. Brown, *Fourier Series and Boundary Value Problems*, 3rd ed., McGraw-Hill, 1978.
2. R. N. Bracewell, *The Fourier Transform and Its Applications*, 2nd ed., McGraw-Hill, 1978.
3. T. G. Beckwith, R. D. Marangoni and J. H. Lienhard V, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995.
4. W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flanner, *Numerical Recipes - The Art of Scientific Computing*, 2nd ed., Cambridge University Press, 1992.

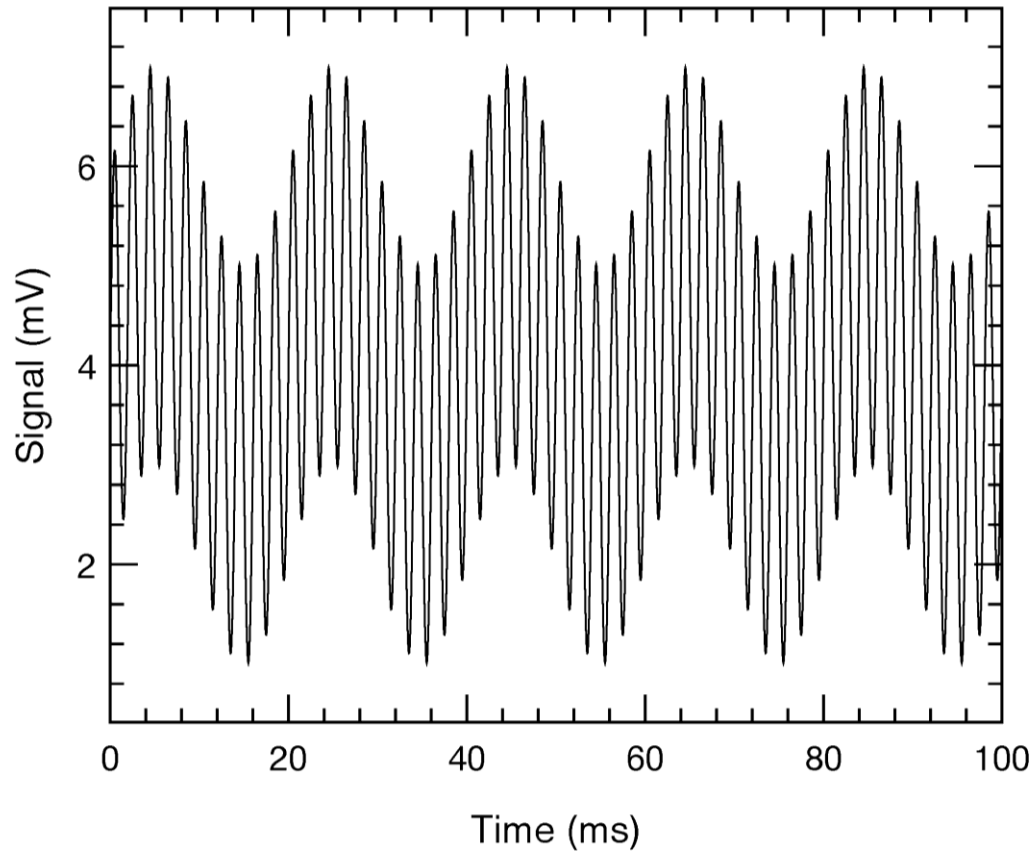


Figure 1. A waveform composed of a fundamental mode (at 50 Hz) and its 9th harmonic (at 10 times the fundamental frequency, or 500 Hz). The waveform also has a DC, or time-averaged, component of 4 mV. Specifically, the signal (in millivolts) is $4 + \sin(100\pi t) + 2\sin(1000\pi t)$, or equivalently, based on cosines, $4 + \cos(100\pi t - \pi/2) + 2\cos(1000\pi t - \pi/2)$, which simply represents a phase shift of $-\pi/2$.

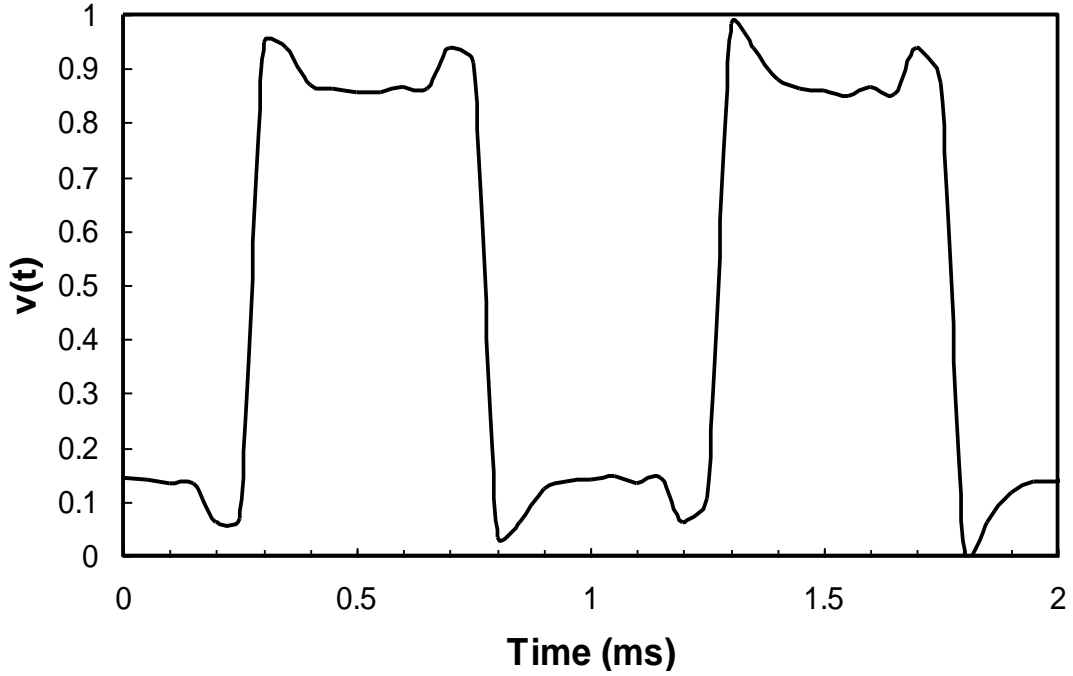


Figure 2. Partial reconstruction of a square wave using five sine waves, each with a different amplitude, frequency and phase, and a separate DC component. The individual waves are shown in Fig. 3.

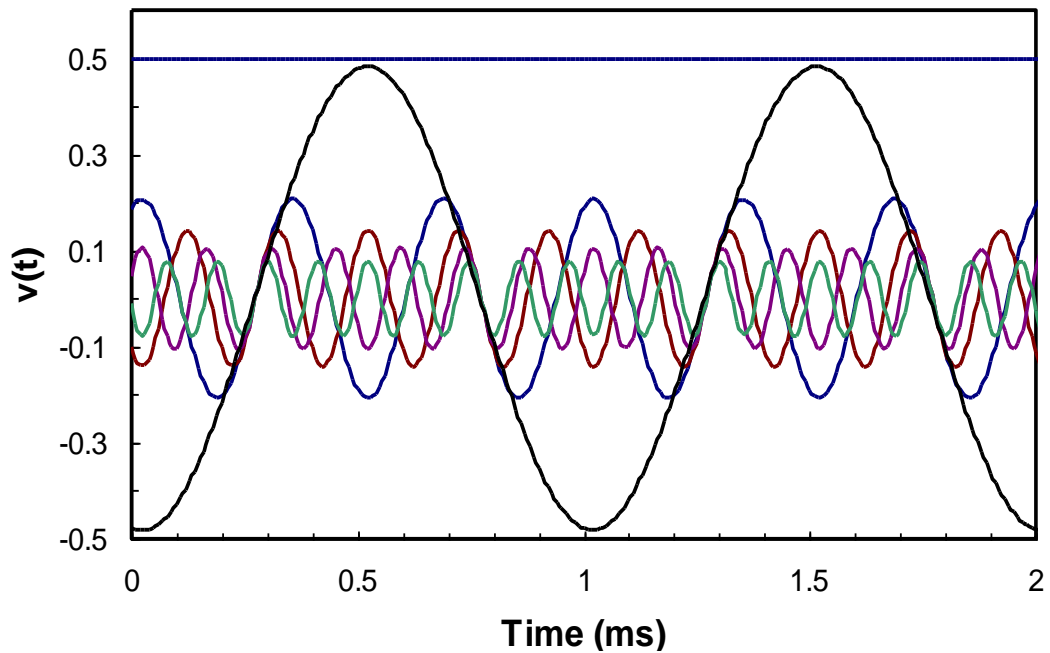


Figure 3. The five sine waves and constant function used to construct the square wave shown in Fig. 2.

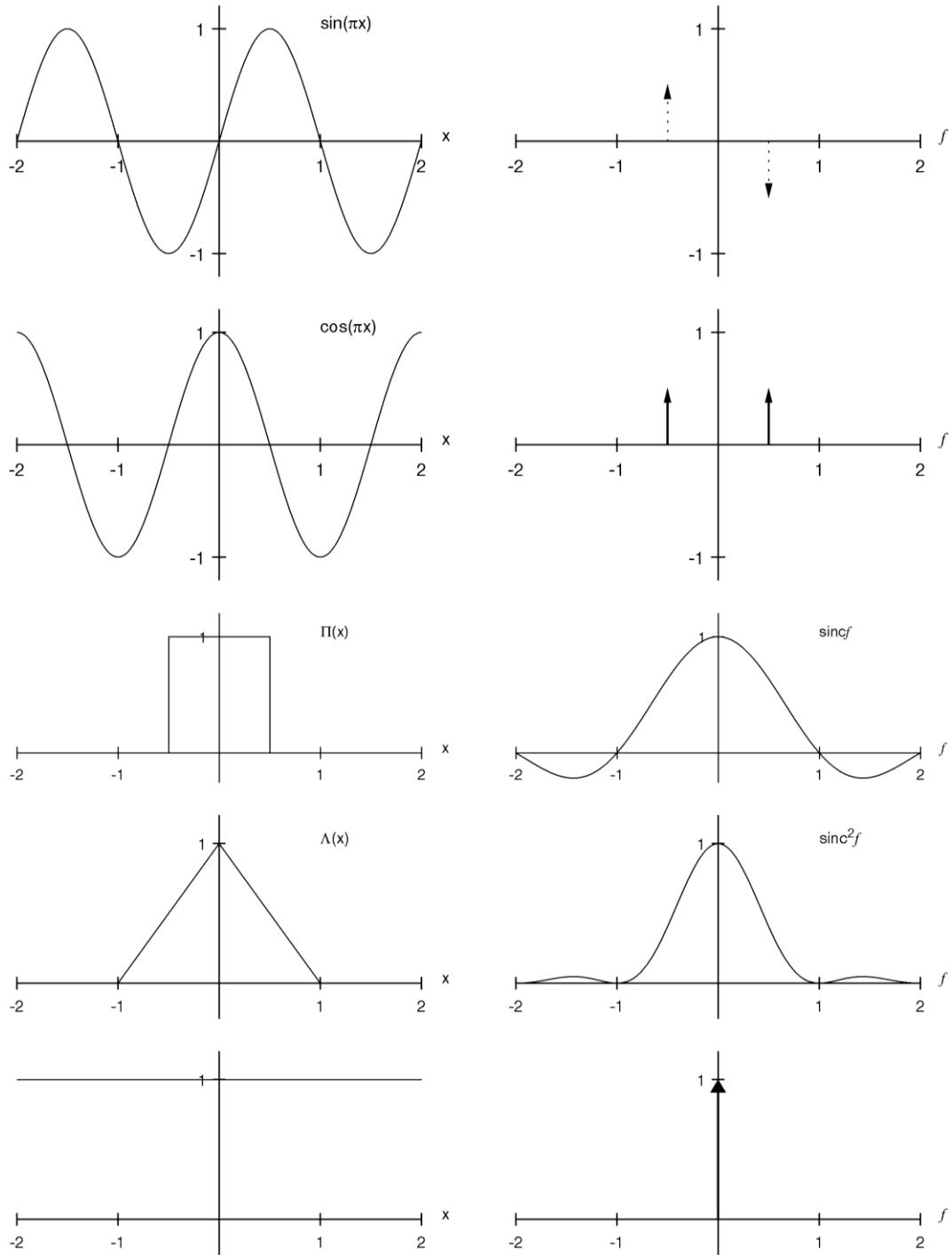


Figure 4. Fourier transforms of various functions (left and right pairs). The arrows represent impulse functions (i.e., delta functions), which extend infinitesimally along the x -axis, but have a integrated area corresponding to the height indicated by the arrow. The dashed regions indicate imaginary values.

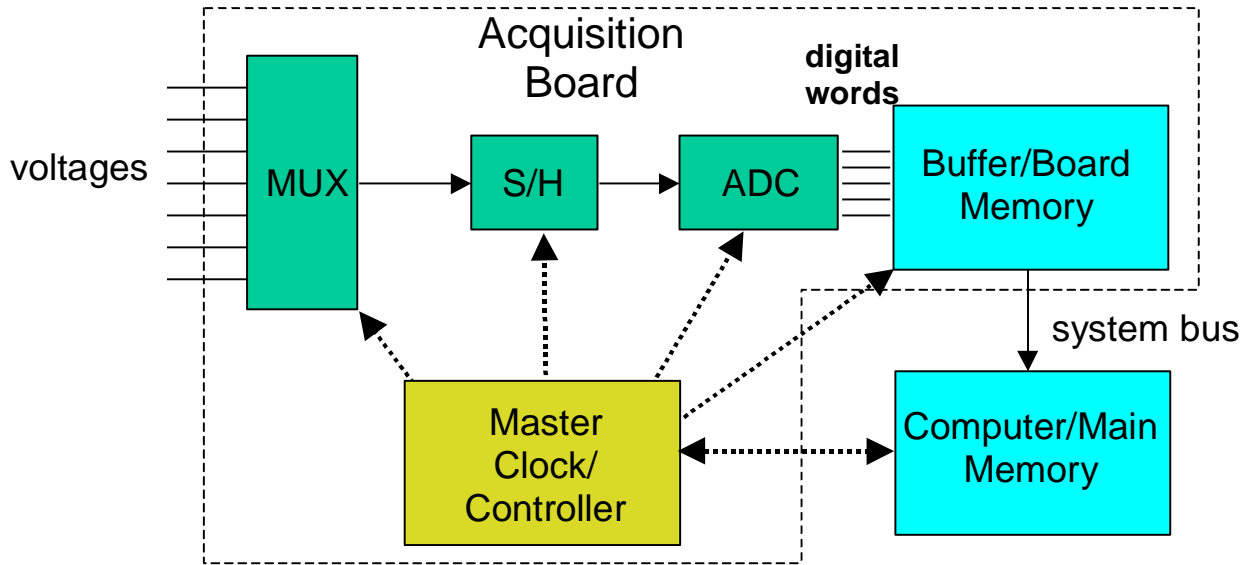


Figure 5. Schematic of multiplexed, sequential sampling, computer data acquisition board and its connection to the computer.

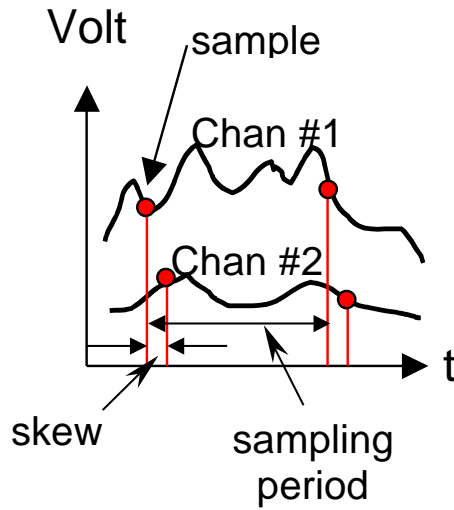


Figure 6. Time delay (skew) between successive channels in sequential sampling system.

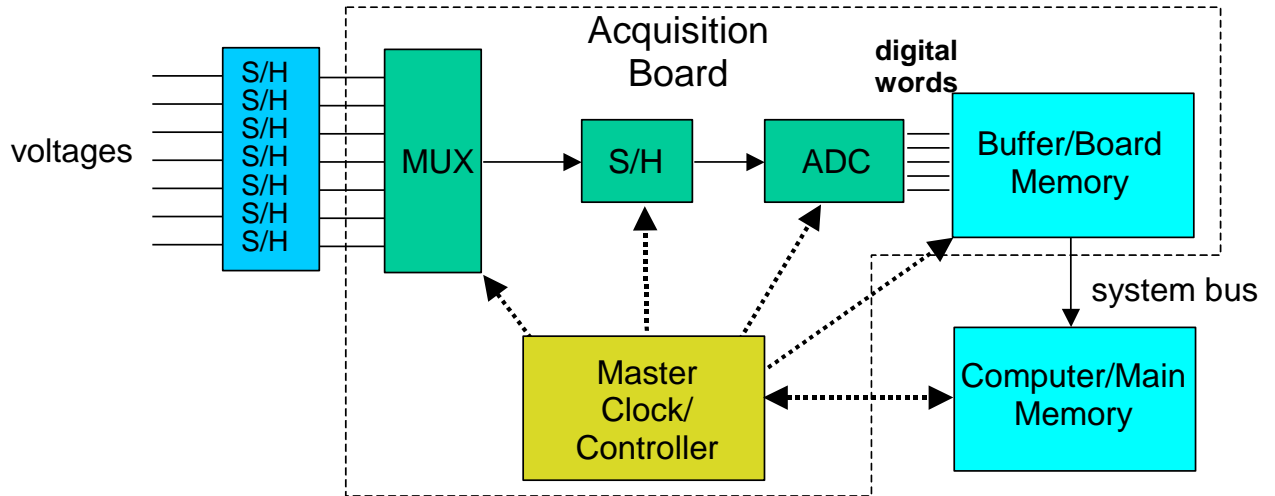


Figure 7. Schematic of simultaneous sampling, computer data acquisition system.